Polarised AdS Black Holes

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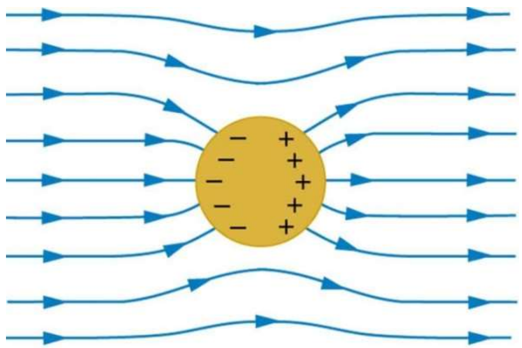
Outline

- Objectives and Motivation
- Einstein-Maxwell-Λ theory
  - Exact solutions
  - Thermodynamics
  - AdS/CFT
- Numerics
  - De Turck method
- Results
  - Deformed horizon
  - Phase diagram
- Current work

Motivation

- Study gravitational physics in AAdS spaces
  - Gravitational box
  - Gauge/gravity duality (QCD and CMT applications)
    - Phase transitions (deconfinement, superconductivity, etc.)
    - Time dependent phenomena (thermalization)

- Polarized black hole
  - Study thermodynamics (construct phase diagram)
  - Generalize HP phase transition
  - Dual CFT interpretation



Einstein-Maxwell-Λ

- Eintein-Hilbert action with cosmological constant

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left(R - 2\Lambda - \frac{1}{4} F^2 \right)$$

- Use variational principle to determine equations of motion

$$R_{\mu\nu} - \frac{R}{4} g_{\mu\nu} = 2T_{\mu\nu}$$
$$\nabla^\mu F_{\mu\nu} = 0$$

$$T_{\mu\nu} = F_{\mu\alpha} F_\nu^\alpha - \frac{1}{4} g_{\mu\nu} F^2$$

Einstein-Maxwell-Λ

- Some exact solutions to the equations

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega_2^2$$

  - AdS Space

$$f(r) = 1 + \frac{r^2}{L^2}$$

  - Schwarzschild-AdS black hole

$$f(r) = 1 + \frac{r^2}{L^2} - \frac{2M}{r}$$

  - Reissner-Nordstrom-AdS black hole

$$A = \left(\frac{Q}{r} - \frac{Q}{r_0}\right)dt$$

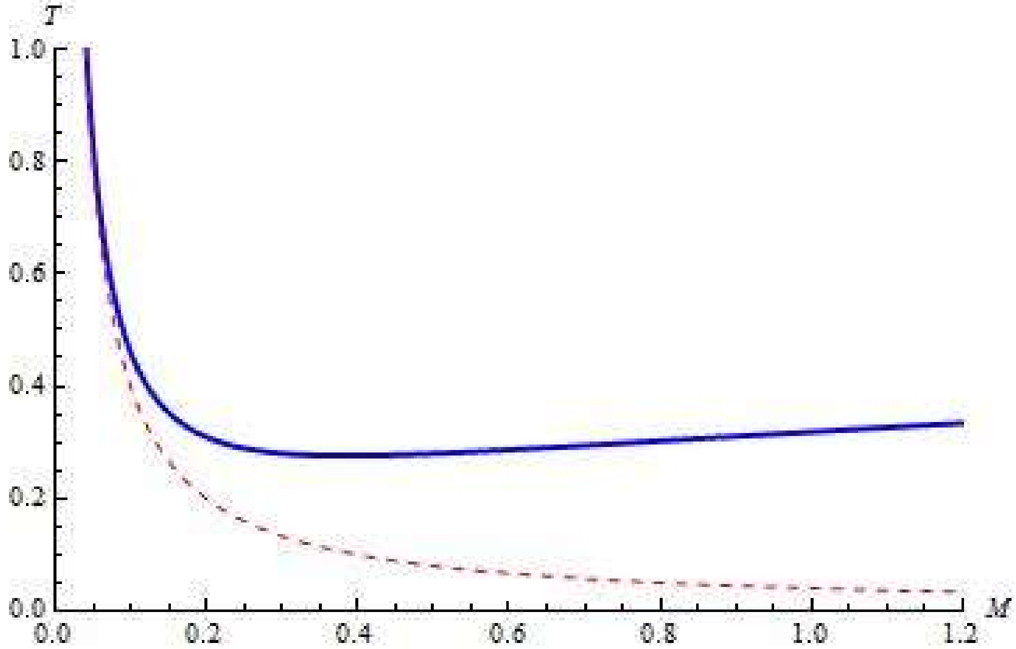
$$f(r) = 1 + \frac{r^2}{L^2} - \frac{2M}{r} + \frac{Q^2}{r^2}$$

Thermodynamics

- Expanding the metric near the horizon $r = r_0$

$$ds^2 = dq^2 + \left(\frac{f'(r_0)}{2}\right)^2 q^2 d\tau^2$$

  - To avoid conical singularities at the black hole surface, τ has to be periodic with period $\beta = T^{-1} = 2\pi \frac{2}{f'(r_0)}$
  - Schwarzschild-AdS: $T = \frac{L^2 + r_0^2}{4\pi \mathrm{r}_0 L^2}$ (solid line)
  - Schwarzschild: $T = \frac{1}{4\pi \;{}_0}$ (dashed line)



Thermodynamics

- Partition function

$$Z = \int D\phi \; e^{-I[\phi]} \approx e^{-I}$$

- Free energy determines the favoured state

$$F = -T \log Z \approx TI$$

- Thermodynamical quantities can be derived from the partition function

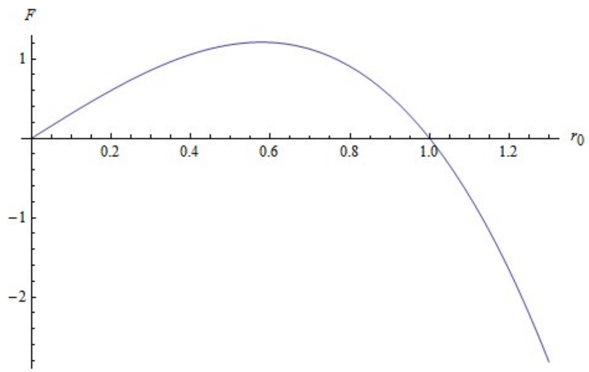
$$< E > = -\frac{\partial Z}{\partial \beta} \qquad\qquad S = \beta < E > + \log Z$$

Hawking-Page phase transition

- Compare free energies to determine thermodynamically favoured state

$$F = M - TS$$

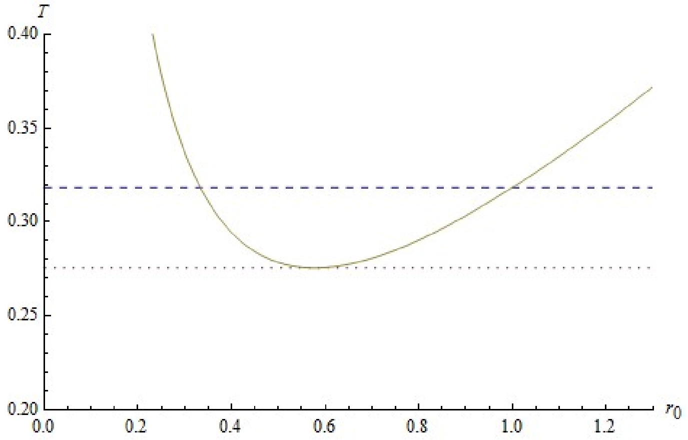
- $M = \frac{r_0}{2}(1 + r_0^2)$
- $T = \frac{1+3r_0^2}{4\pi \;_0}$
- $S = \pi r_0^2$



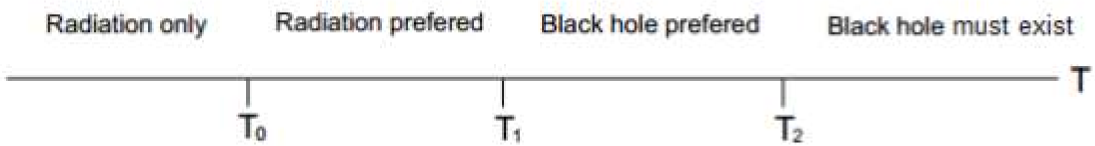
- There is a value of r_0 (or T) for which the thermodynamically favoured state changes from pure AdS to Schwarzschild Ads → Hawking-Page phase transition

Hawking-Page phase transition

- Temperature of a Schwarzschild AdS black hole, with critical temperature (dashed)



- One dimensional phase diagram



Dual CFT perspective

- Bulk field $\phi \leftrightarrow O_\phi$ boundary operator

- IR divergencies (long distance) in the gravitational side $\leftrightarrow$ UV divergencies in the field theory

- To regulate divergencies near the boundary, introduce Fefferman-Graham coordinates (adequate for expansions near $z = 0$)

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + g_{ij}(z,x)dx^i dx^j\right)$$

$$g_{ij}(z,x) = g_{ij}^{(0)}(x) + z g_{ij}^{(1)}(x) + z^2 g_{ij}^{(2)}(x) + \cdots$$

Holographic renormalization

- Regulate the theory by moving boundary to $z = \epsilon$ then subtract divergent terms as $\epsilon \to 0$

- Expectation value for an operator O_ϕ in the presence of sources (value of the bulk field ϕ at the boundary)

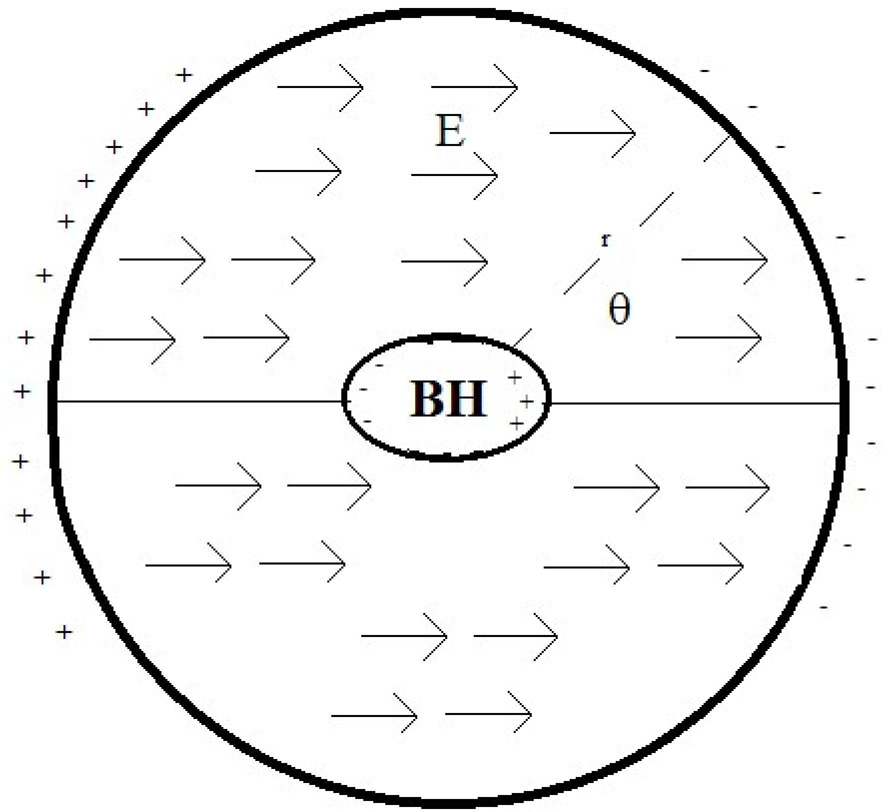
$$< O_\phi > = \frac{1}{\sqrt{g^{(0)}}} \frac{\delta S_{ren}}{\delta \phi^{(0)}}$$

- Metric can be seen as a background source for boundary stress energy tensor

$$< T_{ij} > = \frac{3L}{16\pi G} \; g_{ij}^{(3)}$$

Polarized black hole

- Breaking of spherical symmetry, but azimuthal (ϕ) symmetry is maintained
- Polarize introducing boundary conditions at conformal boundary
- Also can be done for a space without BH (AdS Soliton)



Einstein-de Turck

- Einstein-Maxwell equations are not fully elliptic , but there is still some freedom in the choice of coordinates

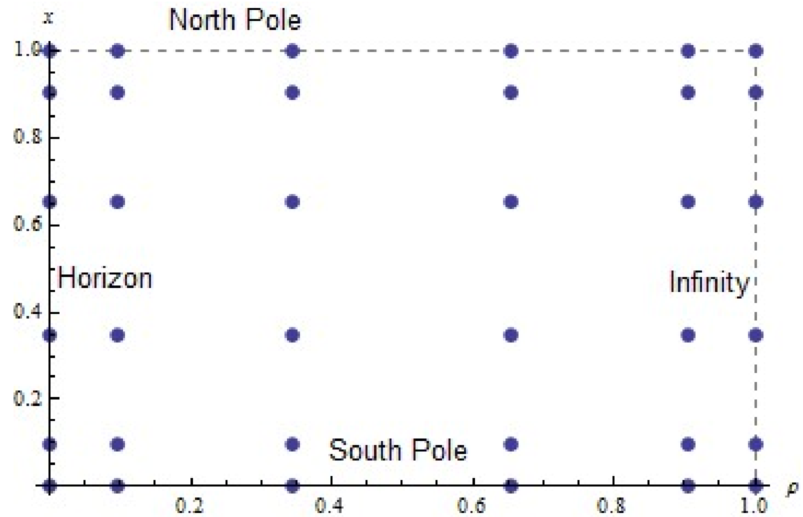
- Introduce the de Turck vector ξ

$$\xi^\alpha = g^{\mu\nu}(\Gamma^\alpha_{\mu\nu} - \bar{\Gamma}^\alpha_{\mu\nu})$$

- $R_{\mu\nu} \to R_{\mu\nu} - \nabla_{(\mu}\xi_{\nu)}$

- Introduce a new set of coordinates adequate to the discretization
  - $\rho^2 = 1 - \frac{r_0}{r}; \quad \sin\theta = 4x(1-x)$

- Use spectral methods in a Chebyshev grid



Einstein-de Turck

- Reference metric can be Schwarzschild-AdS
- The ansatz (for the metric and gauge field) should allow for breaking of spherical symmetry
- Boundary conditions
  - Smoothness at the poles
  - Regularity at the horizon
  - Metric should go to AdS at the conformal boundary (infinity)
- In the final solution, the norm of the de Turck vector $\sqrt{\xi^{\alpha}\xi_{\alpha}}$ should be zero

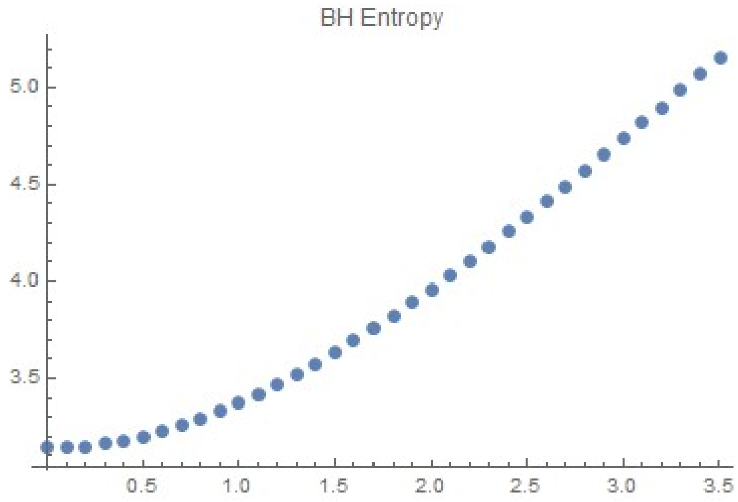
Deforming the solution

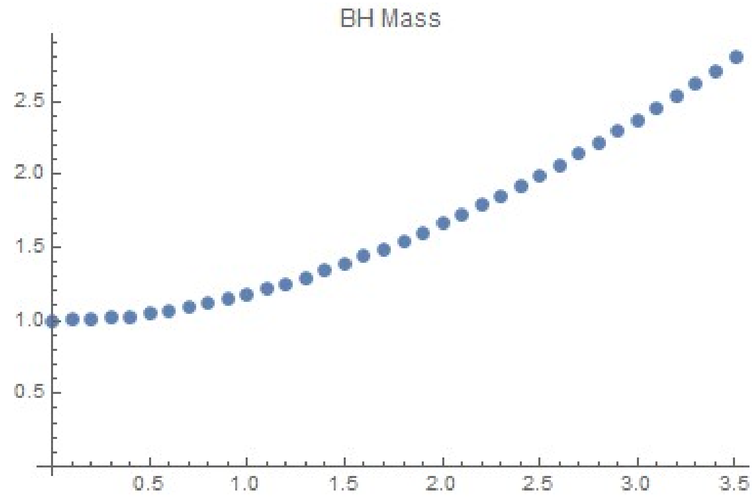
- Introduce boundary conditions for the gauge field that break spherical symmetry

$$A = \frac{F_6(\rho, x)\rho^2}{r_0} dt$$

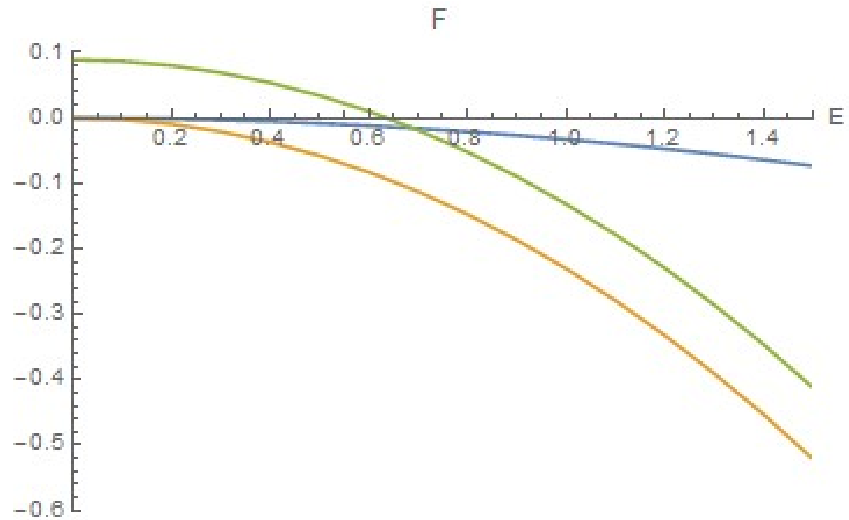
  - $F_6(1, x) = 0$ represents the Schwarzschild case
  - $F_6(1, x) = \Phi$ represents the Reissner-Nordstrom case with electromagnetic potential Φ
  - $F_6(1, x) = E \cos\theta$ represents the case of a space dependent potential which we argue is the same as an external electric field

Results



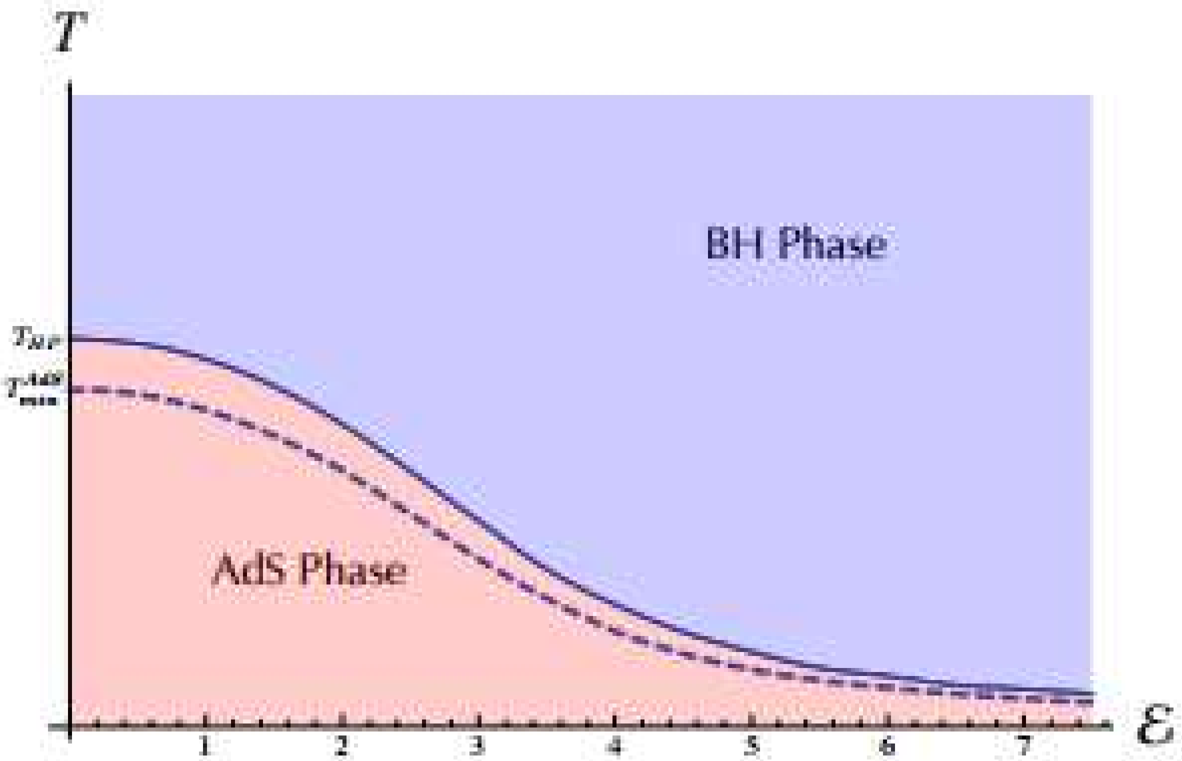


- BH phase becomes favoured where the lines intersect



Results

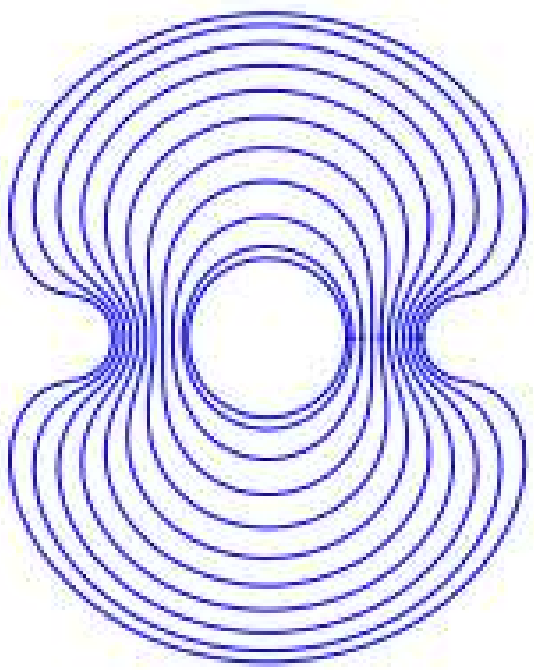
- Generalized Hawking-Page phase transition

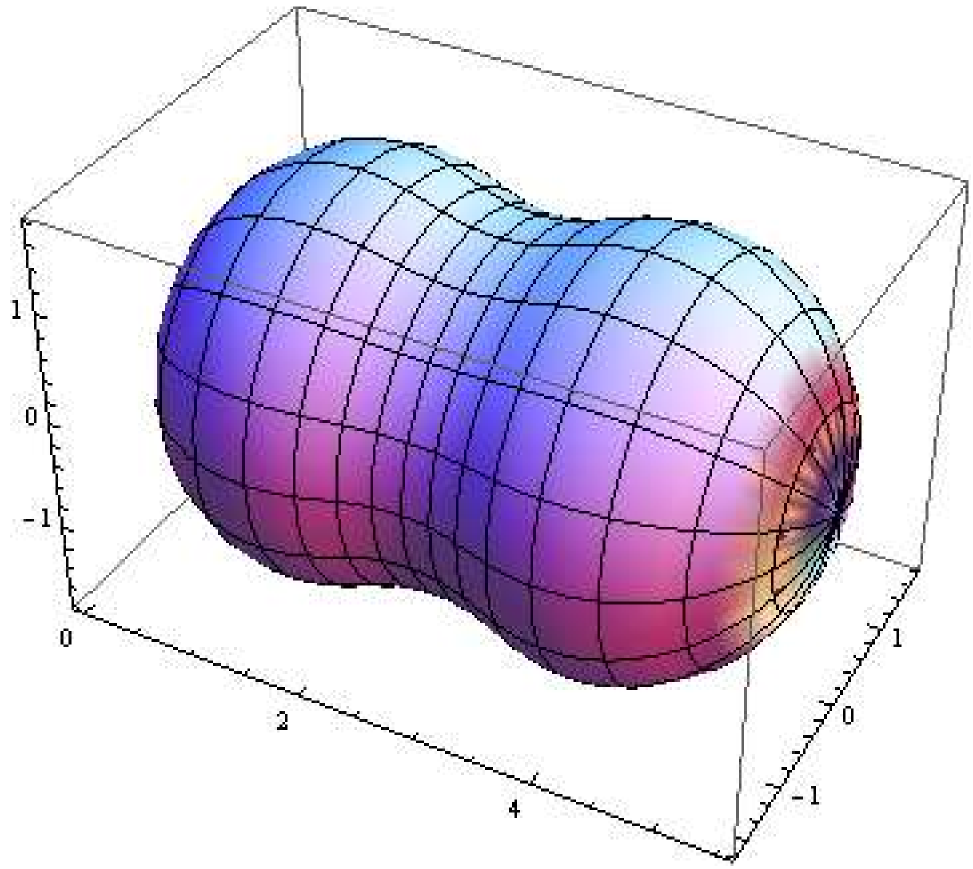


arXiv:1511.08505 [hep-th]

Results

- Solutions for the case with an applied electric field exhibit deformations of the black hole surface, as shown in the isometric embeddings into R^3 of the horizon

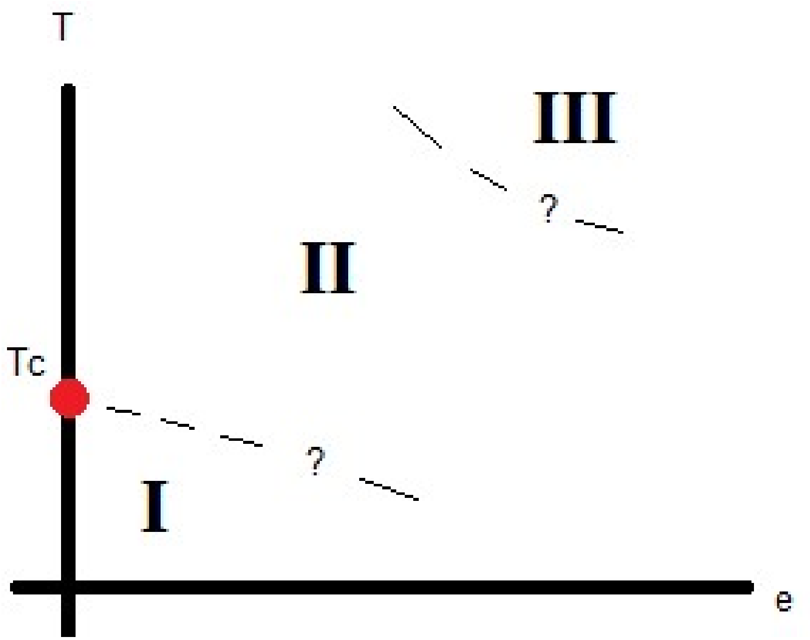




Current work

- Construct an accurate phase diagram
  - I – AdS space with electric flux
  - II – Deformed (polarized) AdS black hole
  - III – Two oppositely charged black holes

- Construct 2BH solution
  - AAdS space with two equal and oppositely charged black holes
  - Requires patching



Thank you