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# A very short introduction to BRST quantization and the Gribov gauge fixing ambiguity


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# Overview

Quantization of QCD: BRST symmetry, unitarity and renormalization

Gribov copies and how to deal with them in the path integral

(Softly broken) standard BRST

BRST symmetric formulation of Gribov-Zwanziger theory in Landau gauge

Comparison with gauge fixed lattice theory

Generalization to the linear covariant gauge

# Quantum chromodynamics

## Some definitions

- We study strong (color) interaction between gluons and quarks.
- The classical action is

$$S_{YM} = \int \mathrm{d}^4 x \left( \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \overline{\psi}_f i \not{D} \psi_f \right)$$

with

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \\ D_\mu &= \partial_\mu - i g A^a_\mu t^a \end{aligned}$$

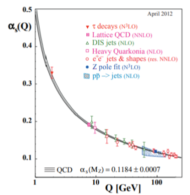
- Gluons described by (massless) gauge fields $A^a_\mu$, 6 flavours of (quasi-)massless and heavier (anti)quarks by $\overline{\psi}_f / \psi_f$, each in 3 colors.

# Quantum chromodynamics

## Some observations

- *All* particles carry color charge, also the gluon force carriers ($\neq$ uncharged photon in electrodynamics)
- Another big difference: at low energies, QCD is strongly interacting, while at high energies, it is perturbatively accessible

$$\text{asymptotic freedom: } g^2(\text{energy}) \sim \frac{1}{\ln(\text{energy}/\Lambda_{QCD})}$$




- We expect much richer structure for QCD than for QED!

# Quantum chromodynamics (at zero temperature)

## 2 crucial "effects"

- confinement: no free color charges (quarks/gluons), but only color-neutral states (mesons, baryons, glueballs, hybrids)


- issue of chiral symmetry breaking: QCD action has chiral symmetry (massless quarks)
  $\Rightarrow$ dynamically this symmetry gets broken
  $\Rightarrow$ quarks get large effective masses, which in return "feed" the mass of the hadrons.
- Why so difficult: inherently nonperturbative problems: effects in $e^{-1/g^2}$

# Quantum chromodynamics (at zero temperature)

## 2 crucial "effects"

- These are in fact "old problems", but still not fully understood/proven!
- What happens at high temperature, relevant for quark-gluon plasma (heavy ion collision programs)!
- For example: why is critical temperature for chiral restoration and deconfinement (almost) the same? How to get analytical estimates for the phase transitions? Link with $T = 0$ physics?

# Computation in QCD

## How to extract physical information from QCD?

- QCD $=$ strongly coupled/nonperturbative physics is important
- hard to handle analytically
- several options:
  1. model building I: e. g. AdSQCD aka Holographic QCD (see other talks here)
  2. model building II: e. g. effective models like NJL models, . . . (see other talks here)
  3. computer simulations: lattice QCD (see other talks here)
  4. quantize the theory and try to get as good information as possible by variety of techniques

# Computation in QCD

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  1. model building I: e. g. AdSQCD aka Holographic QCD.
  2. model building II: e. g. effective models like NJL models, ...
  3. computer simulations: lattice QCD
  4. quantize the theory and try to get as good information as possible by variety of techniques $=$ our method
  5. compare with lattice data the intermediate and final results

# Computation in QCD

## Lattice simulations

- Lattice QCD: gauge invariant expectation values can be computed without gauge fixing
- Space-time is discretized
- Use Monte Carlo simulations with the partition function $\int [dA] e^{-S_{YM}}$
- The numerical estimates are in good agreement with experimental data
- Comment: Real-life QCD (with massless/massive quarks) is however not that easy to simulate...
- Also difficult for real-time applications (transport properties etc).

# QCD


## Our goal?

- We wish to start from “elementary” knowledge: the basic ingredients of YM: gluon & ghost (& quark) propagators and the interactions (vertices) between them
- We shall work in a specific gauge, and try to describe/understand nonperturbative effects
- These basic ingredients then enter in studies of spectrum, phase transition, etc.

# QCD

## Our goal?

- QCD is a gauge theory, so physics should be gauge invariant (gauge independent).
- QCD is also strongly coupled $\rightarrow$ nonperturbative machinery indispensable
- Continuum methods require partial modelling of nonperturbative interactions $\rightarrow$ how well is gauge covariance under control?
- This is our main motivation, since most continuum methods rely on Landau (or to lesser extent Coulomb) gauge. Nowadays, lattice QCD is also starting to offer insights into other gauges.

# Faddeev-Popov quantization of QCD

## Classical level (quarks omitted)

- Classical $SU(N)$ Yang-Mills action in $d=4$ Euclidean space time

$$S_{YM}=\frac{1}{4}\int \mathrm{d}^4x F^a_{\mu\nu}F^a_{\mu\nu}$$

- $S_{YM}$ possesses enormous local invariance,

$$A_\mu \quad \rightarrow \quad A^S_\mu \;=\; S^+\partial_\mu S+S^+A_\mu S \qquad S\in SU(N)$$

  or in infinitesimal form

$$A^a_\mu \quad \rightarrow \quad A^a_\mu+D^{ab}_\mu\omega^b \qquad D^{ab}_\mu \;\equiv\; \partial_\mu\delta^{ab}-gf^{abc}A^c_\mu$$

- Gauge symmetry $\leftrightarrow$ massless spin 1 particles
- Freedom to choose gauge

# Faddeev-Popov quantization of QCD

## The Yang-Mills action

- In quantum field theory $\Rightarrow$ path integral formalism easiest way to quantize/compute correlation functions with partition function

$$Z_{FP} = \int [dA] e^{-S_{YM}}$$

- This path integral is ill-defined, we need to gauge fix

# Faddeev-Popov quantization of QCD

## Why is the path integral ill-defined?

- Take only the quadratic part ($\rightarrow$ free propagator)

$$\int [dA] \exp\left[\int d^4x - \frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu) + J^a_\mu A^a_\mu + \dots\right]$$

rewrite this as

$$Z_{FP} = \int [dA] \exp\left[-\frac{1}{2}\int \mathrm{d}^4x A^a_\nu \underbrace{\left(\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu\right)}_{K_{\mu\nu}} A^a_\nu + J^a_\mu A^a_\mu + \dots\right]$$

- Similar to Gaussian integral

$$\int dX \exp\left[-\frac{1}{2}x^T A x + J^T x\right] \quad \sim \quad (\det A)^{-1/2} \exp\left[\frac{1}{2}J^T A^{-1} J\right]$$

- Inverse of $K_{\mu\nu}$ needed $\Rightarrow$ does not exist due to zero modes (no gluon propagator?)

# Faddeev-Popov quantization of QCD

## The Yang-Mills action

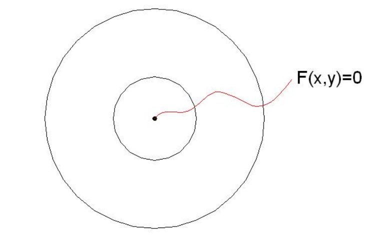
- In quantum field theory $\Rightarrow$ path integral formalism easiest way to quantize/compute correlation functions

$$Z_{FP} = \int [dA] e^{-S_{YM}}$$

- This path integral is ill-defined
- Due to the "enormous" gauge symmetry $\Rightarrow$ overcounting. Zero modes related to pure gauge modes: $(\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)(\partial_\nu\phi) = 0$.
- We must choose a gauge
- Physically, this does sound reasonable, as gauge equivalent configurations describe the same physics.

# Faddeev-Popov quantization of QCD

**The gauge orbits: set of every $A_\mu^a$ and all of its gauge equivalent configurations**

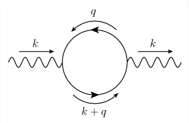



On every gauge orbit, pick out a single gluon field as solution of certain condition $F(A) = 0$.

# Faddeev-Popov quantization of QCD

## Which gauge to choose?

- In principle, complete freedom
- Select a gauge suitable for the specific problem under study
- However: important restriction
  the eventual gauge fixed theory needs to be renormalizable
  $\Rightarrow$ quantum corrections contain (UV) divergences, that need to be consistently handled/removed by rescaling parameters/fields in the action. This is called renormalization (see later).




$$\sim \int d^4q \frac{1}{q^2(q+k)^2} = \infty$$

- Not all gauges are renormalizable!

# Faddeev-Popov quantization of QCD

- Faddeev and Popov implemented a linear covariant gauge choice as follows

$$\begin{aligned} Z_{FP} &= \int [dA]\delta(\partial A - B)\det \mathcal{M}^{ab} e^{-S_{YM}} \\ \mathcal{M}^{ab} &= -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A_\mu^c) \quad = \text{Faddeev-Popov operator} \end{aligned}$$

- The FP trick is based on the functional version of

$$\int dx \delta[f(x) - y] g(x) = \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x)=y}$$

- Notice that this *assumes* that $f(x) = y$ only has a single solution! Otherwise one needs

$$\int dx \delta[f(x) - y] g(x) = \sum_i \left\{ g(x) \left| \frac{\partial f}{\partial x} \right|^{-1} \right\}_{f(x_i)=y}$$

# Faddeev-Popov quantization of QCD

## The Faddeev-Popov action

- Faddeev and Popov implemented a gauge choice as follows

$$Z_{FP} = \int [dA] \underbrace{\delta(\partial A - B) \det \mathcal{M}^{ab}}_{\to \text{ unity}} e^{-S_{YM}}$$

- $\delta(\partial A) \Rightarrow \partial A = 0 \equiv$ Landau gauge if $\alpha = 0$
- Faddeev-Popov determinant $\det \mathcal{M}^{ab}$ is corresponding Jacobian
- This form is not suitable to work/compute with (we want Feynman rules from local action)

# Faddeev-Popov quantization of QCD

## The Faddeev-Popov action in the Landau gauge

- We shall work with linear covariant gauge $\partial_\mu A^a_\mu = \alpha b^a$.
- Very popular gauge, as it has many nice (quantum) properties.
- The eventual gauge fixed action reads

$$S_{YM} + S_{gf} \quad = \quad \int \mathrm{d}^4 x \left( \frac{1}{4} F^2_{\mu\nu} + b^a \partial_\mu A^a_\mu - \frac{\alpha}{2} b^a b^a + \overline{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

- Gaussian sampling over $B$ via $\int [dB] e^{-\frac{B^2}{2\alpha}}$, introduction of the $b$-field multiplier and anti-commuting (ghost) scalars for the lifting of the det into the exponential.

# Degrees of freedom in pure QCD

## classical level

- 4 polarizations per $A_\mu^a$ in $d = 4$
- one can use classical gauge invariance to kill 2 polarizations
- Eventually: 2 physical (transverse) polarizations (analogous to QED)

# Degrees of freedom in pure QCD

## quantum level

- what about the $b-$ and (anti)ghost fields?
- no more gauge symmetry to kill 2 gluon degrees of freedom?
- so, what is particle content?
- Also, we lost “theory-defining gauge symmetry”. So what does gauge symmetry mean at the quantum level?

# The BRST symmetry

## an important (crucial) symmetry

- Quantum action enjoys nilpotent BRST symmetry, $s(S_{YM}+S_{gf})=0$

$$sA_\mu^a=-D_\mu^{ab}c^b\,,\qquad sc^a=\frac{g}{2}f^{abc}c^bc^c$$
$$s\bar{c}^a=b^a\,,\qquad sb^a=0\,,\qquad s^2=0$$

- Quantum replacement for classical gauge invariance
- Used for proofs of
  - (perturbative) renormalizability via *Slavnov-Taylor identity*
  - (perturbative) unitarity: e.g. ghost, antighost, longitudinal and time-like gluon polarizations cancel, only 2 transverse gluon degrees of freedom survive!
  - Gauge independence of physical observables.
  - BRST symmetry is an important concept

# BRST Application 1: perturbative unitarity (Kugo-Ojima)

- Define $|\psi_p\rangle$ physical state $\Leftrightarrow Q_{BRST}|\psi_p\rangle = 0$, $|\psi_p\rangle \neq Q_{BRST}|\ldots\rangle$ using *asymptotic* Fock states/free BRST charge. Do no forget nilpotency: $Q^2_{BRST} = 0$. So if $|\psi_u\rangle = Q_{BRST}|\omega\rangle$, then $\langle\psi_u|\psi_u\rangle = 0$ and $\langle\psi_p|\psi_u\rangle = 0$.
- $|\psi_p\rangle \in \mathrm{cohomology}(Q_{BRST})$
- unphysical states $|\psi_u\rangle$ always come in quartets, and decouple from the physical spectrum.
- Indeed: If $\mathcal{N}$ = counting operator of longitudinal and temporal gauge polarization, ghost, antighost, then $\mathcal{N} = \{Q_{BRST}, \mathcal{R}\}$
- If $\mathcal{N}|\psi_p\rangle = n|\psi_p\rangle$ (with $n \neq 0$), then automatically $|\psi_p\rangle = \frac{1}{n} Q_{BRST}[\mathcal{R}|\psi_p$
- So, we find a trivial (zero norm) state, so no unphysical particles in subspace! The 2 transverse gluons remain, with positive norm (as is checked explicitly, this is not following from the BRST).
- BRST analysis thus guarantees perturbative unitarity ($Q_{BRST}$ commutes with Hamiltonian $H$ as symmetry)

# BRST Application 2: gauge independence of gauge invariant quantities

- We rewrite

$$S_{YM+GF} = S_{YM} + \int d^4x\, s\left(\overline{c}^a \partial^\mu A^a_\mu + \frac{\alpha}{2} b^a \overline{c}^a\right)$$

- Consider a gauge invariant operator, $s\mathcal{O} = 0$. Then

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\alpha}\langle \mathcal{O}\rangle &= \frac{\mathrm{d}}{\mathrm{d}\alpha}\int [\mathrm{dfields}]\mathcal{O}e^{-S_{YM+GF}} \\
&= \int [\mathrm{dfields}]\mathcal{O}s\left(\frac{1}{2}b^a\overline{c}^a\right)e^{-S_{YM+GF}} \\
&= \int [\mathrm{dfields}]s\left[\mathcal{O}\left(\frac{1}{2}b^a\overline{c}^a\right)\right]e^{-S_{YM+GF}} \\
&= \langle s(\ldots)\rangle = 0
\end{aligned}$$

- More complicated but also BRST based: gauge independence of e.g. electron mass, although corresponding to pole of gauge variant fermion propagator (Nielsen identities).

# BRST Application 3: proof of renormalizability

- QFT with classical action $\Sigma(A, g)$ ($A =$ field, $g =$ parameter)
- As said before, quantum corrections are contaminated with divergences.
- Renormalization process: if we can deform action as

$$\Sigma(A, g) + \delta\Sigma(A, g) \stackrel{?}{=} \Sigma(A_0, g_0)$$

  so that
  - $\Sigma(A, g) + \delta\Sigma(A, g)$ gives finite correlation functions, i.e. the (divergent) counterterms in $\delta\Sigma(A, g)$ cancel the divergences coming from $\Sigma(A, g)$
  - The counterterms can be reabsorbed in a (infinite) rescaling of the *already present* original action fields and/or parameters via

$$A_0 = Z_A A\,, \qquad g_0 = Z_g g$$

    (otherwise you loose predictability)
- The bare (divergent) quantities like $g_0$ are not measurable, the renormalized (finite) quantities like $g$ are.

# BRST Application 3: algebraic renormalization

- Consider a QFT with a certain symmetry of the action $\Sigma$, expressed in a functional way by $\mathcal{F}(\Sigma) = 0$ (Ward identity).
- One can prove that the quantum corrected action obeys the same symmetries as the original action[1].
- The symmetry is putting a constraint on the form of the most general quantum correction, $\Sigma_c$. If we can solve for the most general quantum correction which obeys $\mathcal{F}(\Sigma_c) = 0$, it is a straightforward exercise to determine whether $\Sigma_c$ can be reabsorbed in the original action by a suitable renormalization of the available parameters. In this way, renormalizability might be obtained to *all* orders of perturbation theory without the necessity of performing complicated Feynman diagram calculations, only allowing to establish renormalizability order by order.

[1] As far as the symmetry is not anomalous.

# BRST Application 3: algebraic renormalization

- The Ward identity arisen from the BRST invariance is known as the Slavnov-Taylor identity, $\mathcal{B}_\Sigma(\Sigma_c) = 0$.
- The most general counterterm (needed to remove the divergences) is thus restricted by $\mathcal{B}_\Sigma \Sigma_c = 0$. Trivial part of the solution:

$$\mathcal{B}_\Sigma \mathcal{B}_\Sigma(\text{anything}) = 0 \,.$$

- Using discrete symmetries and dimension restrictions, it can be shown using cohomological techniques that

$$\Sigma_c = \int d^4x \left[a_0 F_{\mu\nu}^2 + \mathcal{B}_\Sigma(\text{something})\right]$$

  $a_0$ corresponds to renormalization of strong coupling constant $g^2$, the rest renormalizes the gauge fixing etc. Renormalization of $g^2$ is gauge invariant.
- YM theories are renormalizable and thus meaningful, thanks to BRST invariance.

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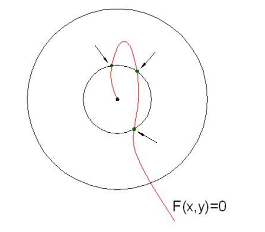
# Potential flaw in FP quantization

## The Gribov problem

- Take $A_\mu$ in linear covariant gauge $\Leftrightarrow \partial_\mu A_\mu = \alpha b$
- Consider (infinitesimal) gauge transform: $A'_\mu = A_\mu + D_\mu \omega$
- $\partial_\mu A'_\mu = \alpha b$ if $\partial_\mu D_\mu \omega = 0$
  GAUGE COPY if FP operator has (normalizable) zero modes!
- Was investigated (and drawn attention to) by Gribov in late seventies for Landau ($\alpha = 0$) and Coulomb gauge ($\partial_i A_i = 0$).

# Potential flaw in FP quantization

## The Gribov problem




- There is still some overcounting when using FP action (which is mathematically seen “wrong”)
- Solution: use the more correct functional version of $\delta$-function where the argument has multiple zeros.
- Unfortunately: cannot be put in useful partition function

# Treating the copy problem

## From Faddeev-Popov to Gribov-Zwanziger: first Landau gauge $\partial A = 0$

- A class of copies was related to zero modes of Faddeev-Popov operator $M = -\partial D$
- Let us restrict path integral to region $\Omega$ where $\partial A = 0$ and $M > 0$. Only sensible if $\partial A = 0$ as only then $M = -\partial D$ is Hermitian!
- $\Omega$ corresponds to local minima of the functional $\int d^4 x A_\mu^2$!
- $\Rightarrow$ This is already an improvement of Faddeev-Popov!
- Compare with lattice where one seeks for (in theory) global minima of $\int d^4 x A_\mu^2$
- How to implement restriction to $\Omega$ in continuum?

# The Gribov-Zwanziger action

## Gribov-Zwanziger

- Gribov and later on Zwanziger worked out this problem and proved many properties of region $\Omega$, together with Dell'Antonio.
- Example: every gauge orbit passes through $\Omega$, it is convex and bounded in every direction ($\longrightarrow$ nice integration region).
- Warning: $\Omega$, region of *relative* minima, is still plagued by Gribov copies. One should work in region of absolute minima, but to my knowledge, not so easy to deal with this.

# Gribov-Zwanziger

## The Gribov restriction: semiclassical level

- Partition function

$$Z = \int [D\Phi] e^{-S_{YM+FP}} \to Z = \int [D\Phi] \theta(M) e^{-S_{YM+FP}}$$

- how to characterize $\theta(M)$? We look at its inverse, schematically written as

$$M^{-1} = (-\partial D)^{-1} = k^2(1 + \sigma(k^2, A))$$

- Gribov no pole condition: $\boxed{\sigma(0, A) < 1}$
- Partition function

$$Z = \int [D\Phi] e^{-S_{YM+FP}} \to Z = \int [D\Phi] \theta(1 - \sigma(0, A)) e^{-S_{YM+FP}}$$

# Gribov-Zwanziger

## The Gribov restriction: semiclassical level

- Semiclassical analysis in thermodynamic limit: $\theta \to \delta$
- Partition function (at the quadratic level) [saddle point evaluation]

$$Z = \int [D\Phi]\delta(1-\sigma(0,A))e^{-S_{YM+FP}} \to \int [D\Phi]e^{-S_{YM+FP}+\gamma^4 \int d^4 x A \frac{1}{\partial^2} A}$$

- $\gamma =$ thermodynamic parameter, fixed by gap equation,

$$\frac{3}{4}g^2 N \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^4+\gamma^4} = 1$$

- New partition function (or action) $\to$ perfect tool to study theory with

# Gribov-Zwanziger

- Restricts the integration to the Gribov region to all orders (Zwanziger, NPB323 (1989), NPB399 (1993))
- The Gribov-Zwanziger action is given by

$$S_{GZ} = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4x \, h(x)$$

with the horizon function

$$h(x) = g^2 f^{abc} A^b_\mu \left(\mathcal{M}^{-1}\right)^{ad} f^{dec} A^e_\mu$$

$$\text{horizon condition } (= \text{gap equation}) \; \langle h(x) \rangle = d(N^2-1)$$

- For $\gamma = 0$, everything reduces to Faddeev-Popov.

# Gribov-Zwanziger

- Somewhat easier derivation than Zwanziger: generalization of Gribov's analysis to all orders Capri et al, PLB719 (2013)
- We considered a classical $A_\mu^a$, and formally computed the inverse via

$$\langle \bar{c}^a(x) c^b(y) \rangle = \frac{\int \mathcal{D}c\mathcal{D}\bar{c}\; \bar{c}^a(x) c^b(y) e^{\int \bar{c}^c \mathcal{M}^{cd} c^d}}{\int \mathcal{D}c\mathcal{D}\bar{c}\; e^{\int \bar{c}^c \mathcal{M}^{cd} c^d}}$$

  Then

$$\mathcal{G}(k,A) = \frac{1}{V(N^2-1)} \langle \bar{c}^a(p) c^a(-q) \rangle|_{p=q=k} = \frac{1}{k^2}(1+\sigma(k,A))$$

  Notice we are *tracing* here, which is a simplification, also present in Zwanziger's derivation.
- At zero momentum, the series can be resummed and leads to

$$\sigma(0,A) = -\frac{g^2}{VD(N^2-1)} \int \frac{d^Dp}{(2\pi)^D} \int \frac{d^Dq}{(2\pi)^D} A_\mu^{ab}(-p) \left(\mathcal{M}^{-1}\right)_{pq}^{bc} A_\mu^{ca}(q)$$
$$= \frac{H(A)}{DV(N^2-1)} .$$

# Gribov-Zwanziger

- No-pole condition: $\sigma(0, A) \leq 1$

$\Rightarrow$ $\theta(1-\sigma) \Rightarrow \delta(1-\sigma) \Rightarrow \langle\sigma\rangle = 1 \Rightarrow \langle H\rangle = DV(N^2-1)$

- So no-pole gives exactly what Zwanziger also obtained, at smaller cost.
- Notice that actually, only a posteriori, we can check that

$$\langle\sigma(k, A)\rangle \leq \langle\sigma(0, A)\rangle = 1$$

i.e. after averaging.

- Likewise,

$$1 + \langle\sigma(k, A)\rangle_{\text{conn}} = \frac{1}{1 - \langle\sigma(k, A)\rangle_{1\text{PI}}}$$

# Gribov-Zwanziger

- We replace the action with a local (equivalent) action

$$S_{GZ} = S_{YM} + S_{gf} + S_h$$

with now

$$\begin{aligned} S_h = & \int d^4x \left( \overline{\varphi}_\mu^{ac} \partial_\nu \left( \partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left( \partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) - g \left( \partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left( D_\nu c \right)^b \varphi_\mu^{mc} \right. \\ & \left. - \gamma^2 g \left( f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g} \left( N^2 - 1 \right) \gamma^2 \right) \right) \end{aligned}$$

- horizon condition ($=$ gap equation)

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \Leftrightarrow \underbrace{\left\langle g f^{abc} A_\mu^a (\varphi + \overline{\varphi})_\mu^{bc} \right\rangle = 2d(N^2-1)\gamma^2}_{d=2\,\text{condensate}}$$

- $\gamma \propto \Lambda_{QCD}$: source of dimensional transmutation.

# Gribov-Zwanziger

## Gribov-Zwanziger quantization

The GZ formalism is a geometrically inspired path-integral construction (cf. boundary condition to stay within the Gribov region in e.g. DSE approach) with good quantum properties that improves upon the standard FP quantization.

## Gribov-Zwanziger quantization

Nice property: closely related to lattice formulation, as in both cases minimization of $\int A^2$ along the gauge orbit is used to define the (a) nonperturbative Landau gauge.

There is the technical issue of having multiple local minima, unknown how to be dealt with at the quantitative level.

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# The Gribov-Zwanziger action

## What about the BRST symmetry?

- The (naturally extended) BRST symmetry

$$s\overline{\omega}_\mu^{ab} = \overline{\varphi}_\mu^{ab} , \qquad s\overline{\varphi}_\mu^{ab} = 0 , \qquad s\varphi_\mu^{ab} = \omega_\mu^{ab} , \qquad s\omega_\mu^{ab} = 0 ,$$

  is softly broken

$$sS_{GZ} = g\gamma^2 \int d^4x \left( f^{abc} A_\mu^a \omega_\mu^{bc} - (D_\mu^{am} c^m)(\overline{\varphi}_\mu^{bc} + \varphi_\mu^{bc}) \right)$$

- Apparently: treating Gribov copy leads to soft breaking of BRST.
- What about gauge parameter independence of correlation functions of BRST invariant operators if we were to generalize GZ to other gauges?

  Let us first find a BRST!

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# Preliminaries

- Consider $A^2$-functional

$$f_A[u] \equiv \mathrm{Tr}\int d^4x\, A^u_\mu A^u_\mu = \mathrm{Tr}\int d^4x \left(u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u\right)^2$$

and set $v = he^{ig\omega}$.

- Working up to 2nd order to identify minima:

$$f_A[v] = f_A[h] + 2\mathrm{Tr}\int d^4x\, \left(\omega\partial_\mu A^h_\mu\right) - \mathrm{Tr}\int d^4x\, \omega\partial_\mu D_\mu(A^h)\omega + O(\omega^3)$$

$\Rightarrow$ $\partial_\mu A^h_\mu = 0 \quad \& \quad -\partial_\mu D_\mu[A^h] > 0$

We recognize the Landau gauge and defining condition of the Gribov region (positive FP operator).

# Preliminaries

- The “minimum configuration” can be solved for

$$A_\mu^h = A_\mu - \frac{1}{\partial^2}\partial_\mu \partial A - ig\frac{\partial_\mu}{\partial^2}\left[A_\nu, \partial_\nu \frac{\partial A}{\partial^2}\right] - i\frac{g}{2}\frac{\partial_\mu}{\partial^2}\left[\partial A, \frac{1}{\partial^2}\partial A\right] + ig\left[A_\mu, \frac{1}{\partial^2}\partial A\right] + i\frac{g}{2}\left[\frac{1}{\partial^2}\partial A, \frac{\partial_\mu}{\partial^2}\partial A\right] + O(A^3)$$

  It is transverse and gauge invariant order by order. See also Lavelle, McMullan, Phys. Rep. 279 (1997).
- Observation: if $\partial A = 0$, $A = A^h$. More precisely

$$A = A^h + \text{non-local power series in } (A, \partial A)$$

# Rewriting GZ action

- $$A = A^h + \text{non-local power series in } (A, \partial A)$$
- Consider GZ action with non-local horizon action

$$H(A) = g^2 \int d^4x d^4y \; f^{abc} A^b_\mu(x) \left[\mathcal{M}^{-1}(x,y)\right]^{ad} f^{dec} A^e_\mu(y)$$

$$\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \, (b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A) \\
&= S_{YM} + \int d^4x \, (b\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c) + \gamma^4 H(A^h) - \gamma^4 R(A)(\partial A) \\
&= S_{YM} + \int d^4x \left(b^h\partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c\right) + \gamma^4 H(A^h)
\end{aligned}$$

with a new field $b^h$

$$b^h = b - \gamma^4 R(A)$$

# Rewriting GZ action

- Introduce auxiliary fields to obtain

$$
\begin{aligned}
S_{GZ} &= S_{YM} + \int d^4x \left( b^h \partial_\mu A_\mu + \bar{c}\partial_\mu D_\mu c \right) \\
&\quad + \int d^4x \ \left( \bar{\phi}\mathcal{M}(A^h)\phi - \bar{\omega}\mathcal{M}(A^h)\omega + \gamma^2 A^h(\bar{\phi}+\phi) \right)
\end{aligned}
$$

  and rename $b^h \to b$ again.

- This new (equivalent) GZ action in the Landau gauge enjoys a nilpotent BRST symmetry

$$
\begin{aligned}
sA_\mu^a &= -D_\mu^{ab} c^b \ , \ sc^a = \frac{g}{2} f^{abc} c^b c^c \ , \ s\bar{c}^a = b^a \ , \ sb^a = 0 \ , \\
s\phi_\mu^{ab} &= s\omega_\mu^{ab} = s\bar{\omega}_\mu^{ab} = s\bar{\phi}_\mu^{ab} = 0
\end{aligned}
$$

  thanks to $sA^h = 0$.

# Rewriting GZ action

- Very nonlocal because of $A^h$ $\to$ hard to discuss renormalizability etc.
- Locality of $A^h_\mu$ via introduction of Stückelberg field

$$A^h_\mu = (A^h)^a_\mu T^a = h^\dagger A^a_\mu T^a h + \frac{i}{g} h^\dagger \partial_\mu h, \quad h = e^{ig\xi^a t^a}$$

- Addition of $\int d^4x \left[\tau \partial A^h - \overline{\eta}\mathcal{M}(A^h)\eta\right]$ to action gives rise to equivalent action as before upon solving the $\xi$-EOM
- $sA^h = 0$ under

$$h \to u^\dagger h\,, \quad h^\dagger \to h^\dagger u\,, \quad A_\mu \to u^\dagger A_\mu u + \frac{i}{g} u^\dagger \partial_\mu u$$

or

$$\begin{aligned} sh^{ij} &= -igc^a (T^a)^{ik} h^{kj} \Rightarrow \\ s\xi^a &= -c^a + \frac{g}{2} f^{abc} c^b \xi^c - \frac{g^2}{12} f^{amr} f^{mpq} c^p \xi^q \xi^r + O(g^3) \end{aligned}$$

# Rewriting GZ action: a few important comments

- $A^h$ is *not* a new field, so no Jacobian etc, as $A^h$ is not an integration variable itself.
- Given any $A$, we *define* $A^h$ as above via its expression in terms of $h$, subject to the constraint $\partial A^h = 0$ via $\tau$-EOM. No question about (non-)uniqueness etc. This definition of $A^h$ gives a controllable (renormalizable) quantum operator.
- If combined with $-\partial D(A^h) > 0$, we have that this $A^h$ corresponds to a local minimum, infinitesimally gauge-connected to $A$ (same logic as GZ-construction, at level of correlation functions).

# Rewriting GZ action

- Can be combined with algebraic renormalization formalism, following Dragon et al, NPB Proc.Suppl.56B (1997). A fully renormalizable framework, see Capri et al, PRD94 (2016) & PRD96 (2017)
- Important comment about renormalization: Addition of $\tau\partial A^h$ saves the day.
  - Standard Stückelberg propagator (via addition of gauge invariant mass term $m^2 A^h A^h$):
  $$\langle\xi\xi\rangle \sim \frac{1}{m^2 p^2}$$
  $\rightarrow$ leads to power counting non-renormalizibility!
  - Here (even for $m \neq 0$):
  $$\langle\xi\xi\rangle \sim \frac{1}{p^4}$$
- Final (important) point of interest

$$\langle A \ldots \overline{c} \ldots c\rangle_{\text{new GZ}} \equiv \langle A \ldots \overline{c} \ldots c\rangle_{\text{old GZ}} \qquad \text{(Landau gauge)}$$

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# The Refined Gribov-Zwanziger action in linear covariant gauge: extra dynamical effects

- We include extra dynamical effects due to non-perturbative gauge invariant $d=2$ condensates $\rightarrow$ lower vacuum energy Dudal et al, PRD78 (2008), PRD84 (2011) & work in progress
- Ghost propagator $G(p^2) \sim \frac{1}{p^2}$ for $p^2 \sim 0$, $G(p^2) \neq \frac{1}{p^4}$.
- Gluon propagator

$$D(p^2) \propto \frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4}, \qquad D(0) \neq 0\,.$$

$m^2$ and $M^2$ are mass scales corresponding to condensates, in particular

$$m^2 \sim \left\langle A^h A^h \right\rangle, \qquad M^2 \sim \langle \overline{\varphi}\varphi - \overline{\omega}\omega \rangle$$

- Works pretty well to describe Landau lattice data Oliveira et al, arXiv:1803.02281; PRD81 (2010) 074505; Cucchieri al, PRD85 (2012) 094513; Bornyakov et al, PRD85 (2012) Rodriguez-Quintero et al, PRD88 (2013) . Massive-type gluon propagator with *cc* poles.

  This is of course nice, but also not fully satisfactory: there is still a (direct) dependence on lattice input.

# The Refined Gribov-Zwanziger action in Landau gauge

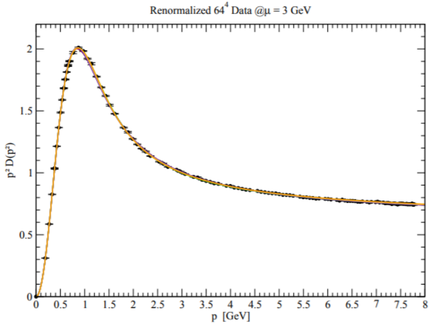


Figure: Figure taken from Dudal, Oliveira, Silva, arXiv:1803.02281 [hep-lat]. Fit to lattice data with $D(p^2) = Z\frac{p^2+M^2}{p^4+(M^2+m^2)p^2+\lambda^4}$. This results in propagator having 2 complex conjugate poles at $p^2 \approx -0.25 \pm 0.45i$ GeV$^2$. The gluon is thus not a physical propagating particle in this picture. For more about unphysical ("confined") gluons, see talk of Roelfs.

# Lattice gluon propagator in the linear covariant gauge

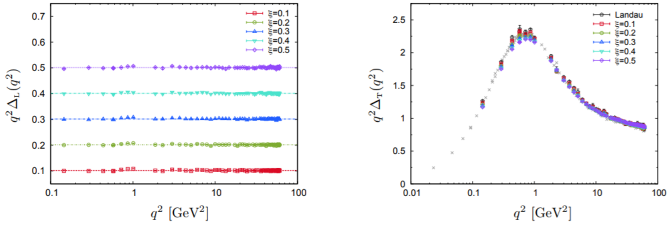


Figure: taken from Bicudo et al, PRD92 (2015)

We notice almost no dependence on gauge parameter. Nonperturbatively, the longitudinal propagator still fixed by $\frac{\alpha}{p^2}\frac{p_\mu p_\nu}{p^2}$.

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# How to tame Gribov copies in generic linear covariant gauge

- We will restrict to

$$\Omega^h = \{ A^a_\mu | \, \partial_\mu A^a_\mu = -i\alpha b^a, \, \mathcal{M}^{ab}(A^h) > 0 \}$$

  Due to $\partial A^h = 0$, $\mathcal{M}^{ab}(A^h) > 0$ makes sense.
- In Dudal et al, PRD92 (2015), it was shown that this removes all infinitesimal gauge copies that are connected via Taylor expansion in $\alpha$ to Landau gauge zero modes.
- The (BRST symmetric) RGZ action reads

$$\begin{aligned} S \quad = \quad & S_{YM} + \int d^4x \left( \alpha \frac{b^a b^a}{2} + i b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right) \\ & + \int d^4x \, \tau^a \partial_\mu (A^h)^a_\mu + \int d^4x \bar{\eta}^a \partial_\mu D^{ab}_\mu(A^h) \eta^b + \int d^4x \frac{m^2}{2} A^{h,a}_\mu A^{h,a}_\mu \\ & + \int d^4x \left( -\bar{\varphi}^{ac}_\mu \mathcal{M}(A^h)^{ab} \varphi^{bc}_\mu + \bar{\omega}^{ac}_\mu \mathcal{M}(A^h)^{ab} \omega^{bc}_\mu + g\gamma^2 f^{abc} (A^h)^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) + M^2 (\bar{\varphi}^{bc}_\mu \varphi^{bc}_\mu - \bar{\omega}^{bc}_\mu \omega^{bc}_\mu) \right) \end{aligned}$$

# How to tame Gribov copies in generic linear covariant gauge

Sketch of the argument

- From

$$\partial A = \alpha b \Rightarrow A = A^h + \tau \quad \text{with} \quad \partial\tau = \partial A = \alpha b = \mathcal{O}(\alpha)$$

- Assume that $\zeta$ is such that

$$\partial D\zeta = 0 \qquad \text{(zero mode of standard FP operator)}$$

- Then also

$$\partial D^h\zeta = \partial(\tau\zeta) \Rightarrow \zeta = \left(\partial D^h\right)^{-1}\left(\partial(\tau\zeta)\right)$$

- Linear covariant gauge $=$ continuous deformation around Landau gauge $\alpha = 0$:

$$\zeta = \sum_{n=0}^{\infty} \zeta_n \alpha^n$$

- Recursively,

$$\zeta_n = 0 \ \text{ since } \ \partial D^h = \mathcal{O}(1)\,, \tau = \mathcal{O}(\alpha)$$

# GZ gluon propagator in linear covariant gauge

- One can show that at tree level we still have

$$D(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^4}$$

with (to all orders, BRST based proof!) gauge and renormalization group invariant complex conjugate poles. Suggestive of "small difference" between Landau and other linear covariant gauges; Dudal et al, PRD **95** (2017)

- Slavnov-Taylor identity, next to Ward identity $\frac{\delta\Gamma}{\delta b} = \alpha\partial A$ (gauge condition) allows to prove, also in GZ, that

$$\frac{p_\mu p_\nu}{p^2} \langle A_\mu A_\nu \rangle \equiv \frac{\alpha}{p^2}$$

(consistent with lattice)

- Interesting comment: gluon self energy, $\langle AA \rangle_{1PI}$, is *not* transverse, despite exact BRST invariance. This is only true in absence of propagator mixing!

# GZ ghost propagator in the linear covariant gauge

$$G(k^2) = \frac{1}{k^2} \frac{1}{1-\omega(k^2)},$$

where

$$\omega(k^2) = \frac{Ng^2}{k^2(N^2-1)} \int \frac{d^4q}{(2\pi)^4} \frac{k_\mu (k-q)_\nu}{(k-q)^2} \langle A^a_\mu(q) A^a_\nu(-q) \rangle = \omega^T(k^2) + \omega^L(k^2)$$

$\omega^T(k^2)$ comes from transverse component of gluon propagator, with

$$\omega^T(k^2) = c + \mathcal{O}(k^2)$$

$\omega^L(k^2)$ stems from the (standard perturbative) longitudinal component, at 1-loop

$$\omega^L(k^2) = \alpha \frac{Ng^2}{64\pi^2} \log \frac{k^2}{\bar{\mu}^2}$$

as we have exactly $D_{L,\mu\nu}(k^2) = \frac{\alpha}{k^2} \frac{k_\mu k_\nu}{k^2}$. So, logarithmic IR suppression is there, and will remain to be there at higher orders.

# Lattice ghost propagator in the linear covariant gauge

- No predictions yet, because the FP operator $-\partial D$ is non-Hermitian for $\alpha \neq 0 \rightarrow$ numerically not so easy nut to crack how to invert this FP operator.
- Work in progress by Roelfs, Oliveira, Silva et al, based on lattice linear covariant gauge minimizing functional Cucchieri & Mendes, PRL103 (2009), Binosi et al, PRD92(2015).

$$\min_U \mathrm{Tr} \int d^4x \left( A_\mu^U A_\mu^U + \frac{2}{g} \mathrm{Re}(iU\Lambda) \right)$$

  $\Lambda^a$ Gaussian sampled with width related to gauge parameter.
- In conjunction with Gribov copy question based on this functional.

# The End!

# Thanks!