The gauge/gravity duality and the pomeron

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- Regge Theory: study of analytical properties of scattering as a function of complex angular momentum

$$\mathcal{A}(s,t) \sim s^{j(t)};$$

- $j(t)$ is a function fixed by the spectrum of the particles being exchanged with given quantum numbers: Regge trajectory;
- If the particles have the quantum numbers of the vaccum the Regge trajectory is known as the pomeron;
- Soft ($j_0 = 1.09$) and Hard ($j_0 = 1.4$) pomerons have been successful in describing successfully the elastic pp and Deep Inelastic Scattering (DIS) processes;
- How are the soft and hard pomeron related? Are they the same object?

- The gauge/gravity duality is a tool to study strongly coupled Quantum Field Theories using a classical weakly coupled gravitational theory in a higher dimensional space-time;
- Fields in the bulk have normalizable and non-normalizable modes

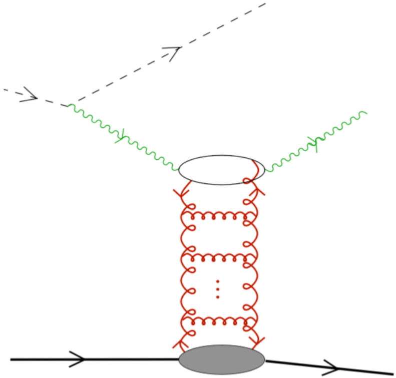
$$\phi(z,x) \sim \phi_0 z^{\Delta_-} + \phi_+ z^{\Delta_+}$$

- Operator insertion is dual to the non-normalizable mode of the dual field;
- Expectation values are dual to the normalizable mode of the dual field;
- The pomeron trajectory is dual to the graviton's Regge trajectory.

$$Z_{QFT}[J] = e^{-S_{on-shell}}$$

- $\gamma^* p \to X$, sum over final states X
- $F_2\left(Q^2, x\right) = \frac{Q^2}{4\pi^2\alpha}\frac{1}{s} Im\mathcal{A}\left(s, t=0\right)$
- Hadronic tensor:

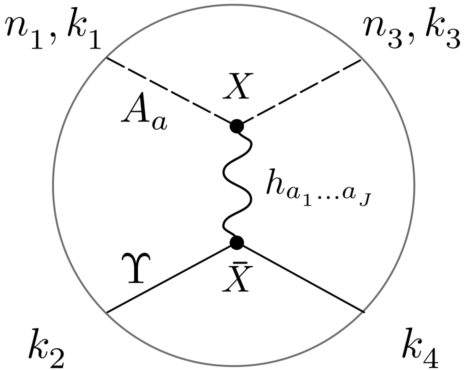
$$W^{\mu\nu} = i\int d^4 x e^{iq\cdot x}\left\langle H, P\right|\left[J^{\mu}\left(x\right), J^{\nu}\left(0\right)\right]\left|H, P\right\rangle$$



- Holographic dual of large N_c Yang-Mills theory:

$$S = M_P^3 N_c^2 \int d^5x \sqrt{-g} e^{-2\Phi} \left[R + 4 (\partial\Phi)^2 + V(\Phi) \right]$$

- $ds^2 = e^{2A(z)} \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right)$, $\Phi = \Phi(z)$ dual to the energy-momentum tensor and lagrangian respectively;
- Spacetime coordinates (x_i, z). z is associated with the inverse of the 4D RG scale;
- $\lambda = e^\Phi$ is identified with the running 't Hooft coupling;
- The potential is fixed in order to reproduce the β funciton of the field theory.


n_1, k_1
n_3, k_3
X
A_a
$h_{a_1 \ldots a_J}$
Υ
$\bar{X}$
k_2
k_4

- Proton target: normalizable modes of a scalar dual to $|H, P\rangle$

$$\Upsilon(x, z) = e^{iP\cdot x}\upsilon(z) \; ;$$

- J^{μ} act like sources so they are dual to non-normalizable modes of a $U(1)$ gauge field in the bulk

$$S_A = -\frac{1}{4}\int d^5X\sqrt{-g}\, e^{-\Phi}\left(F_{ab}F^{ab} + \beta R_{abcd}F^{ab}F^{cd}\right) \; ;$$

- Second term motivated by the fact that there are only two possible tensor structures allowed by causality. Higher order corrections from string theory.

- $A_\mu^\lambda(X;k,\lambda) = n_\mu(\lambda)\, f_k(z)\, e^{ik\cdot x}$

$$\left[-Q^2 + e^{\Phi-A}\partial_z\left(e^{A-\Phi}\partial_z\right) + \beta\Delta_\beta\right] f_Q(z) = 0\,,$$
$$\Delta_\beta = -2e^{-2A}\left[\left(-\dot{A}\ddot{A} - \dot{\Phi}\ddot{A} + \dddot{A}\right)\partial_z + \ddot{A}\partial_z^2 - \dot{A}^2 Q^2\right]\;;$$

- The pomeron is dual to the spin J fields in the graviton Regge trajectory;
- In the Regge limit only the TT components of these fields matter;
- Linearize action to get vertices with the graviton and generalize it to spin J.

- Non-Minimal coupling $U(1)$ gauge field

$$\beta_J \int d^5X\sqrt{-\bar{g}}e^{-\Phi}\Big[F^{z\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_{J-1}}F^{\alpha_J}{}_z\mathcal{D}^J_{\parallel}+$$
$$F^{\mu\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_{J-1}}F^{\alpha_J\nu}\left(e^{2A}\mathcal{D}^J_{\perp}\eta_{\mu\nu}+\partial_\mu\partial_\nu\right)+$$
$$2F^{\mu\alpha_1}\partial^{\alpha_2}\cdots\partial^{\alpha_{J-1}}F^{\alpha_J z}\left(\partial_z-J\dot{A}\right)\partial_\mu\Big]\,h_{\alpha_1\cdots\alpha_J}\,,$$
$$\mathcal{D}^J_{\perp}=e^{-2A}\dot{A}\left(\partial_z-(J-2)\,\dot{A}\right),$$
$$\mathcal{D}^J_{\parallel}=e^{-2A}\left(\partial_z^2-(2J-1)\,\dot{A}\partial_z-(J-2)\,\ddot{A}+J\,(J-1)\,\dot{A}^2\right).$$

- Minimal coupling scalar field

$$\bar{\kappa}_J\int d^5X\sqrt{-\bar{g}}\,e^{-\Phi}\,\left(\Upsilon\partial^{\alpha_1}\ldots\partial^{\alpha_J}\Upsilon\right)\,h_{\alpha_1\ldots\alpha_J}\,.$$

$$\left[\nabla^2 - 2e^{-2A}\dot{\Phi}\nabla_z - \frac{\Delta(\Delta-4)}{L^2} + J\dot{A}^2 e^{-2A} + \right.$$
$$\left. + (J-2)\left(a\ddot{\Phi} + b\left(\ddot{A} - \dot{A}^2\right) + c\dot{\Phi}^2\right)\right] h^{TT}_{\alpha_1\cdots\alpha_J} = 0$$

- a,b,c are fitting parameters;
- Compatible with the graviton EOM and with the AdS case;
- Second term comes from tree level coupling of a closed string;
- All terms of dimension inverse squared length compared with the first two points are included;

$$\frac{\Delta(\Delta-4)}{L^2} = \frac{2}{l_s^2}(J-2)\left(1 + \frac{d}{\sqrt{\lambda}}\right) + \frac{1}{\lambda^{4/3}}\left(J^2 - 4\right)$$

- d is a constant and l_s is a length scale set by the QCD string.

- In the Regge limit the $+\cdots+$ component decouples from the other components;

$$\left\{\left[\partial_z + 2\dot{A} - 2\dot{\Phi}\right]\left[\partial_z - \dot{A}\right] + \nabla_\perp^2 - m^2(z)\, e^{2A}\right\} e^{(1-J)A} h^{TT}_{+\cdots+} = 0$$

- Integrating over the light-cone coordinates

$$\int \frac{dw^+ dw^-}{2} \Pi_{+\cdots+,-\cdots-}(w, z, \bar{z}) = -\frac{i}{2^J} e^{(J-1)(A+\bar{A})} G_J(z, \bar{z}, l_\perp)$$

- $G_J(z, \bar{z}, l_\perp)$ is a propagator on the transverse space of the scattering process and satisfies

$$\left[\Box_3 - 2\dot{\Phi}\partial_z - e^{-2A}\left(2\dot{A}^2 + \ddot{A} - 2\dot{A}\dot{\Phi}\right) - \Delta(\Delta - 4)\right] G_J = -e^{2\Phi}\delta^3(x, x')$$

- The resulting EOM can be transformed in a 1-d quantum mechanics problem;

$$G\left(z, l_{\perp}\right)=e^{iq\cdot x}e^{\Phi-A/2}\psi(z), \quad \left[\partial_z^2+t-V(z)\right]\psi(z)=0\,;$$

- The solution for the propagator is then

$$G_J(z,\bar{z},t)=e^{\Phi+\bar{\Phi}-\frac{A+\bar{A}}{2}}\sum_n \frac{\psi_n(J,z)\,\psi_n^*(J,\bar{z})}{t_n(J)-t}\,;$$

- The scattering amplitude with exchange of a spin J field is then

$$A_J\left(s,t\right)=\frac{\beta_J\bar{\kappa}_J s^J}{2^J}\int dzd\bar{z}e^{-\Phi-\bar{\Phi}-2J(A+\bar{A})+3A+5\bar{A}}\upsilon_m^2(\bar{z})$$
$$\times\left(f_Q^2(z)\mathcal{D}_{\perp}^J+\frac{\dot{f}_Q^2(z)}{Q^2}\mathcal{D}_{\parallel}^J\right)\left[e^{(J-1)(A+\bar{A})}G_J(z,\bar{z},0)\right].$$

- We need to sum over all even spin J fields $J \geq 2$;
- Sommerfeld-Watson transform

$$\frac{1}{2}\sum_{J\geq 2}\left(s^J + (-s)^J\right) = -\frac{\pi}{2}\int \frac{dJ}{2\pi i}\,\frac{s^J + (-s)^J}{\sin \pi J}\,,$$

- Deform the contour from even spin J fields to to the poles $J = j_n(t)$ defined by $t_n(J) = t$;

$$A(s,0) = \sum_n h_n s^{j_n}\int dz\, e^{-(j-2)A+B-\Phi}$$
$$\times\left(f_Q^2\tilde{\mathcal{D}}_\perp^{j_n(0)} + \frac{\dot{f}_Q^2}{Q^2}\tilde{\mathcal{D}}_\parallel^{j_n(0)}\right)\psi_n\big(j_n(0),z\big)\,,$$
$$h_n = -\frac{\pi}{2}\,\frac{\beta_{j_n(0)}\bar{\kappa}_{j_n(0)}}{2^{j_n(0)}}\left(i + \cot\frac{\pi j_n(0)}{2}\right)j_n'(0)$$
$$\times\int d\bar{z}\, e^{\bar{A}(4-j_n(0))}e^{-\bar{\Phi}}e^{\bar{B}}\upsilon_m^2(\bar{z})\,\psi_n^*\big(j_n(0),\bar{z}\big)$$

$$\tilde{\mathcal{D}}_\perp = e^{-2A}\left(\dot{A}\partial_z + \dot{A}^2 + \dot{A}\dot{B}\right),$$

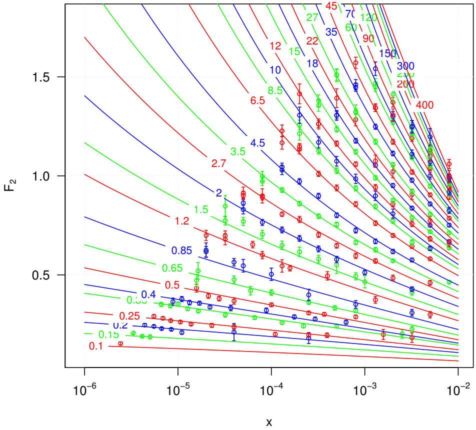
$$\tilde{\mathcal{D}}_\parallel = e^{-2A}\left(\partial_z^2 - (\dot{A} - 2\dot{B})\partial_z + \ddot{B} + \ddot{A} + \dot{B}^2 - \dot{A}\dot{B}\right).$$

- The result is the same as the minimal coupling witht the wave-functions modified by these operators

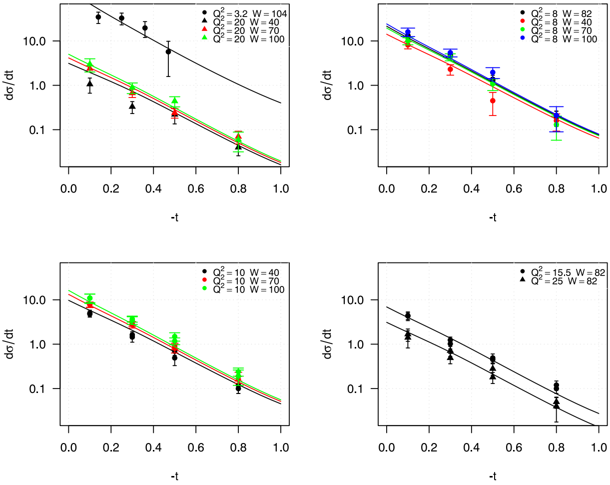
$$F_2(x, Q^2) = \sum_n \left(f_n^{\mathrm{MC}}(Q^2) + f_n^{\mathrm{NMC}}(Q^2)\right) x^{1-j_n},$$

$$f_n^{\mathrm{MC}}(Q^2) = g_n Q^{2j_n} \int dz\, e^{-\left(j_n - \frac{3}{2}\right)A} \left(f_Q^2 + \frac{\dot{f}_Q^2}{Q^2}\right)\psi_n,$$

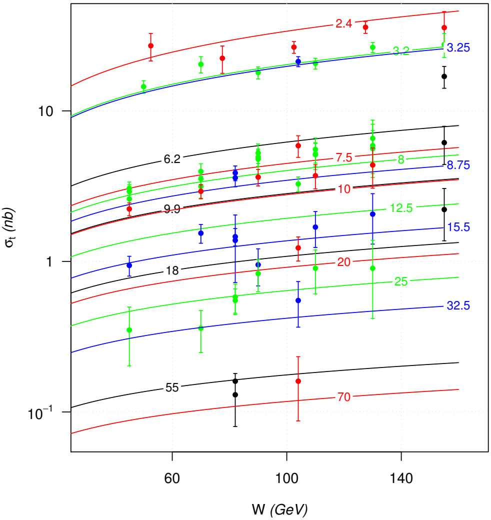
$$f_n^{\mathrm{NMC}}(Q^2) = \tilde{g}_n Q^{2j_n} \int dz\, e^{-\left(j_n - \frac{3}{2}\right)A} \left(f_Q^2 \tilde{\mathcal{D}}_\perp + \frac{\dot{f}_Q^2}{Q^2}\tilde{\mathcal{D}}_\parallel\right)\psi_n.$$



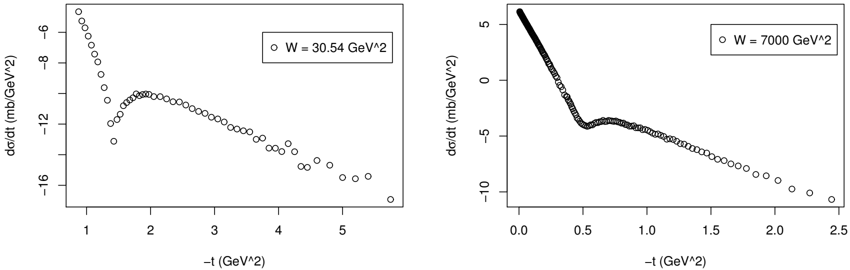
- $\chi^2 = 1.1$, $\chi^2 = 1.7$ just for minimal coupling;
- 13 fittable points explain 240 experimental points;
- Need to test the model in other processes;
- QCD is not able to explain low Q^2 points. We do!



Q2 = 3.2 W = 104
Q2 = 20 W = 40
Q2 = 20 W = 70
Q2 = 20 W = 100
Q2 = 8 W = 82
Q2 = 8 W = 40
Q2 = 8 W = 70
Q2 = 8 W = 100
Q2 = 10 W = 40
Q2 = 10 W = 70
Q2 = 10 W = 100
Q2 = 15.5 W = 82
Q2 = 25 W = 82
dσ/dt
-t
10.0
1.0
0.1
0.0
0.2
0.4
0.6
0.8
1.0



- $\chi^2 = 1.5$ with minimal coupling only;



(a) (b)

Figure: pp scattering at center of mass energie of 30.54 a) and 7000 b) GeV.