$SU(2)_f$ NJL model with Meson Loops: The Gap Equation at Finite Temperature

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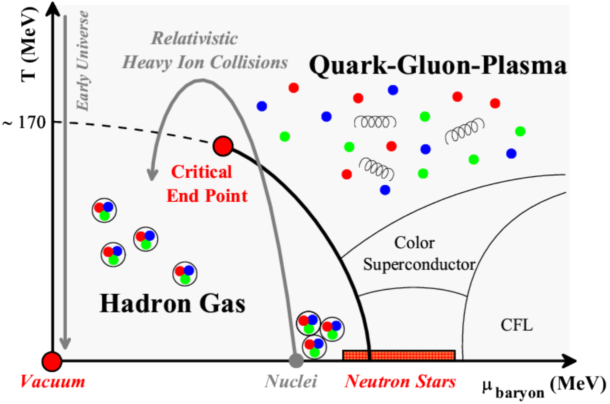
The QCD phase diagram

The different manifestations of QCD matter can be displayed in a $T - \mu_B$ phase diagram.



The theoretical study of the QCD phase diagram can be addressed within different approaches:

- **Lattice QCD**
  - first principle calculations;
  - currently only works on the finite temperature and zero/low density region due to the so called sign problem;

- **Dyson–Schwinger equations**
  - truncation required;

- **Effective models**
  - incorporate the most important features of QCD at a certain energy scale;
  - work on the entire range of the phase diagram;
  - coupling parameters need to be fixed to experimental data or first principle calculations;

The Nambu−Jona-Lasinio model

Widely used as an effective field theory of QCD.

The $SU(2)_f$ NJL Lagrangian density is:

$$\mathcal{L}(\bar{\psi},\psi) = \bar{\psi}(i\partial\!\!\!/ - m)\psi + \frac{g_S}{2}\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\vec{\tau}\psi\right)^2\right].$$

The model is non-renormalizable due to the mass dimension of the coupling in the four-fermi interaction. To regularize the theory we consider a 3-momentum cutoff.

Four parameters to be fixed: Λ_f, m and g_S and a boson cutoff Λ_b.

The NJL gap Equation

When dealing with field theories, the main goal is to calculate the effective action:

$$e^{-\Gamma[\phi]} = \int \mathcal{D}\varphi \exp\left\{-\mathcal{S}\left[\varphi+\phi\right] + \int d^D x \frac{\delta\Gamma\left[\phi\right]}{\delta\phi(x)}\varphi(x)\right\}.$$

This functional generates all the one-particle irreducible (1PI) diagrams of a theory.

Considering small fluctuations around a background field, one can write:

$$\Gamma[\phi] \simeq \mathcal{S}[\phi] \pm \frac{1}{2}\ln\det\left[\frac{\delta^2\mathcal{S}[\phi]}{\delta\phi^2}\right] \ , \ \begin{cases} + & \text{bosons} \ , \\ - & \text{fermions} \ . \end{cases}$$

The *gap* equation of NJL model is usually studied in the mean field approximation.

Only the quark loop contribution is considered.

Background field expansion $\rightarrow$ Calculate next correction

$^1/_{N_c}$ expansion

The first correction are the one-boson loops, which corresponds to a N_c^0 correction.

These corrections can become important at low temperatures due to the pion low mass and $N_c = 3$.

In this work we follow the formalism introduced in (Nikolov et al., 1996; Florkowski and Broniowski, 1996) to derive the one-boson loop *gap* equation.

To get the *gap* equation we calculate the stationary point $\bar{\phi}$ of the one-boson loop effective action.

$$\left.\frac{\delta \Gamma[\phi]}{\delta \phi_a}\right|_{\bar{\phi}} = 0 \;\Rightarrow\; \left.\frac{\delta \mathcal{S}[\phi]}{\delta \phi_a}\right|_{\bar{\phi}} + \frac{1}{2}\,\mathrm{tr}\,\frac{\delta}{\delta \phi_a}\ln\left[\frac{\delta^2 \mathcal{S}[\phi]}{\delta \phi^2}\right]_{\bar{\phi}} = 0.$$

Where only the scalar field is non-zero:

$$\bar{\phi} = (S, 0).$$

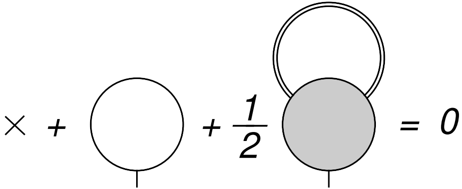
This equation can be written as:

$$\Gamma_0(S) + \frac{1}{2}\Gamma_{0bc}(S)K_{bc}(S) = 0.$$

Here, $\Gamma_{a_1 a_2 \ldots a_n}\left[\bar{\phi}\right]$ is the $n-$quark loop vertex defined as:

$$\Gamma_{a_1 a_2 \ldots a_n}\left[\bar{\phi}\right] = \left.\frac{\delta^n \mathcal{S}[\phi]}{\delta\phi_{a_1}\delta\phi_{a_2}\ldots\delta\phi_{a_n}}\right|_{\bar{\phi}}$$

Diagramatically:



One can then obtain the *gap equation* for the $SU(2)_f$ NJL model:

$$
\begin{aligned}
&\frac{1}{g_S}(S-m) - 4N_cN_fSf_0(S) \\
&+ 2N_cN_fS\int\frac{d^4q}{(2\pi)^4}\left\{4f_1(S,q)+2f_1(S,0)-2\left[q^2+4S^2\right]f_2(S,q)\right\}\tilde{K}_\sigma(S,q) \\
&+ 6N_cN_fS\int\frac{d^4q}{(2\pi)^4}\left\{2f_1(S,0)-2q^2f_2(S,q)\right\}\tilde{K}_\pi(S,q) = 0.
\end{aligned}
$$

One can separate each individual contribution:

$$
G_q(S) + G_\sigma(S) + G_\pi(S) = 0.
$$

The loop functions $f_0(S)$, $f_1(S,q)$ and $f_2(S,q)$ are given by:

$$f_0(S) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+S^2},$$
$$f_1(S,q) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{((k-q)^2+S^2)(k^2+S^2)},$$
$$f_2(S,q) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{((k-q)^2+S^2)^2(k^2+S^2)}.$$

The sigma and pion propagators are:

$$\tilde{K}_\sigma(S,q) = \frac{1}{2N_cN_f f_1(S,0)[q^2+4S^2]},$$
$$\tilde{K}_\pi(S,q) = \frac{1}{2N_cN_f f_1(S,0)q^2}.$$

To include temperature we consider the Matsubara formalism:

$$\int \frac{d^4p}{(2\pi)^4} g(\vec{p}, p_0) = \int \frac{d^3p}{(2\pi)^3} \int \frac{dp_0}{2\pi} g(\vec{p}, p_0) \rightarrow \int \frac{d^3p}{(2\pi)^3} \frac{1}{\beta} \sum_{n=-\infty}^{n=+\infty} g(\vec{p}, \omega_n)$$

Where ω_n are the allowed fermion/boson Matsubara frequencies.

The sum can be carried out by contour integration and residue summation.

$$\frac{1}{\beta} \sum_{n=-\infty}^{n=+\infty} g(\vec{p}, \omega_n) = -\frac{1}{2} \oint \frac{d\omega}{2\pi i} g(\vec{p}, -i\omega) \begin{cases} \tanh\left(\beta\omega/2\right), & \text{fermions} , \\ \coth\left(\beta\omega/2\right), & \text{bosons} . \end{cases}$$

Only the off imaginary-axis poles of the function $g(\vec{p}, -i\omega)$ contribute.

The function $f_2(S, q)$ has poles of order 2. This feature makes the residue calculation quite complicated leading to lengthy expressions.

We can rewrite the $f_2(S, q)$ function as a derivative of the $f_1(S, q)$ function which has only simple poles:

$$f_2(S, q) = -\frac{1}{2}\frac{\partial}{\partial \xi^2} f_1(\xi, q)\bigg|_{\xi = S}.$$

Applying this formalism to the *gap* equation at finite temperature generates **22 different contributions**.

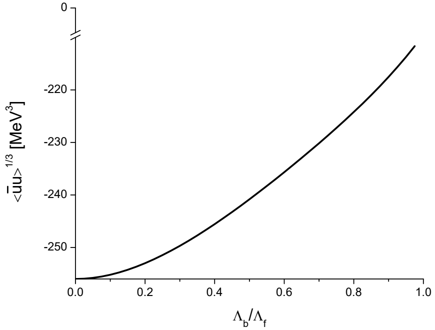
Results

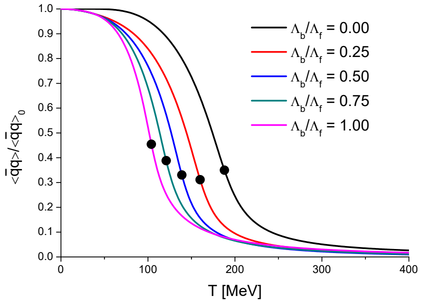
The parameters are fixed to reproduce the value of the quark condensate obtained by two-flavour lattice QCD (Cichy, Garcia-Ramos, and Jansen, 2013), $\left\langle\bar{\ell}\ell\right\rangle^{1/3} = -256$ MeV, the pion mass, $m_\pi = 135$ MeV and the pion decay constant, $f_\pi = 93$ MeV.

The boson cutoff Λ_b, is a free parameter and we take different values to study its effect, yielding different sets of parameters.

The pseudo-critical temperature of the chiral transition is defined through the chiral susceptibility, $\frac{\partial\langle\bar{\ell}\ell\rangle}{\partial T}$.

Λ_b/Λ_f	m[MeV]	Λ_f[MeV]	$g_S^2\Lambda_f$	T_c[MeV]
0.00 (MFA)	4.62	690.32	2.01	188
0.25	4.62	698.60	2.03	160
0.30	**4.62**	**701.78**	**2.04**	**155**
0.50	4.62	717.38	2.07	139
0.75	4.62	740.86	2.13	121
1.00	4.61	770.92	2.21	104





Conclusions

- The inclusion of meson fluctuations brings the pseudo critical temperature of the chiral transition to lower values;
- There is a set of parameters that allows to obtain the pseudo critical temperature of the chiral transition predicted by lattice QCD, $T_c \simeq 155$ MeV;

Further Work

- Include density effects to study the influence of the meson degrees of freedom on the CEP;
- Extend the formalism to include the Polyakov loop allowing the of the deconfinement transition with meson loops;
- Study the critical exponents;
- Study the Equation of State;
- Extend the formalism to $SU(3)_f$;

Thank you for your attention!

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