Critical behavior in gravitational collapse

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Discovery of the critical phenomena

M. Choptuik 1993 [1]

- **One-parameter** families of spherically symmetric spacetimes
- Dynamics driven by a **massless scalar field** $\phi(r,t)$

$$ds^2 = -\alpha^2(r,t)dt^2 + a^2(r,t)dr^2 + r^2 d\Omega^2 \quad (1.1)$$

Initial data

$$\phi(r) = \phi_0 r^3 e^{-\left(\frac{r-r_0}{\delta}\right)^q} \quad (1.2)$$

Time evolution

- Weak members $\Rightarrow$ Flat-space
- Strong members $\Rightarrow$ Black-hole

[1] Choptuik. Phys. Rev. Lett., 70:9, 1993

Critical solution and critical phenomena

Critical solution

The threshold in between both regions is the **critical solution**

- Unique
- Contains a naked sigularity

Subcritical limit

Universal solution characterized by an infinite series of "echoes" in the field profiles, unfolding on incresingly smaller spatiotemporal scales.

Supercritical limit

The black-hole mass scales as a power-law with γ as an apparently universal scaling exponent.

The key result

Collapse of gravitational waves [2]

One-parameter families of initial data in **axisymmetric**, vacuum, general relativity.

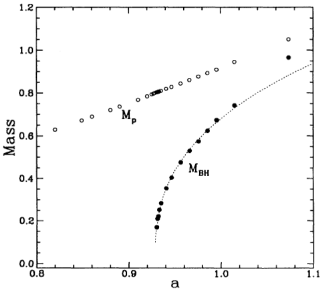

Figure: Filled circles are the critical behavior of the critical black hole mass as a function of the initial wave packet amplitude. Open circles are the quasilocal masses of the wave packets for comparison. The dotted curve is the best fit power law with $a_c = 0.928$, $C = 1.750$, $\beta = 0.369$

$$M_{BH} \simeq C(a - a_c)^{\beta} \tag{1.3}$$

[2] Abrahams and Evans. Phys. Rev. Lett., 70:2980, 1993

The key result

Conclusions and challengues

- This paper proves that the results from M. Choptuik can be extended to the vacuum axisymmetric case
- Reproducing this result has proven difficulty
- We are currently working on fully understand what is happening here and reproducing this result using modern numerical relativity methods.

BAMPS

BAMPS is a pseudospectral code to evolve forward in time axisymmetric gravitational waves [3]

State of the art

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Centered geometrically prolate Brill Waves

$$dl^2 = \gamma_{ij} dx^i dx^j = \Psi^4 \left[e^{2q} \left(d\rho^2 + dz^2 \right) + \rho^2 d\phi^2 \right] \tag{2.1}$$

$$q(\rho, z) = A\rho^2 e^{-[(\rho-\rho_0)^2/\sigma^2)\rho + (z-z_0)/\sigma_z^2]} \tag{2.2}$$

$$A > 0 \quad \sigma_\rho = \sigma_z = 1 \quad \rho_0 = z_0 = 0 \tag{2.3}$$

[3] Hilditch, Weyhausen, Bruegmann. Phys. Rev. D 93, 063006 (2016)

State of the art

Supercritical regime

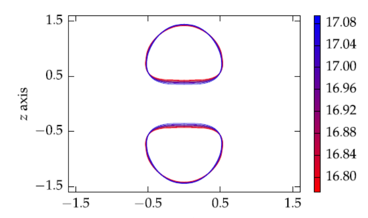

Figure: [4] Two apparent horizons at different times for A = 4.698

- A set of initial data is classified as supercritical when an apparent horizon is found.
- Sets near the critical solution produce axisymmetric binary black holes spacetimes

[4] Hilditch et al. Rev. D 88, 103009 (2013)

Subcritical regime

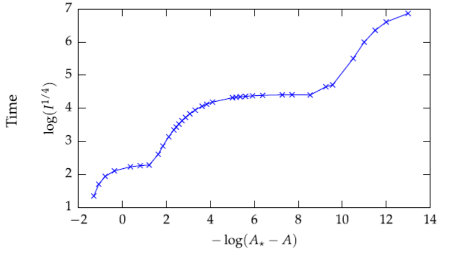

Figure: [4] A* = 4.6966953125

- $I = R_{abcd}R^{abcd} \rightarrow$ Kretschmann scalar
- *Evidence* of power-law scaling of I with the amplitude A
- As only one period can be seen the result is not conclusive

[4] Hilditch et al. Rev. D 88, 103009 (2013)

Workplan: I Systematic comparison

Problems

- There are difficulties in proving previous results
- There are some disagreements among different groups

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Solution

- Collaborations with groups from Prague, Czech and Valencia
- Comparison between the current BAMPS state of the art and the result from them (CoCoNut code and Einstein Toolkit).
- Study first Centered geometrically prolate Brill Waves
- Study and compare different families of initial data (Oblate Brill waves $A < 0$)
- Simulations in full 3D, with no symmetry

Workplan: II Reproduction of results of [2]

Problems

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Challenge

- Theoretical work in order to use the same gauge
- Computing work to deduce the same equations
- Computing the same type of initial data (Teukolsky waves)
- PDE analysis

Future work: Dual-Foliation and Mesh-refinement

Improving BAMPS

- Using dual-foliation approach
- Developing special gauges

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Advantages

- Avoid coordinate singularities in the time evolution
- More efficient use of the computational resources
- Reach higher resolutions
- Simulate critical spacetimes much closer to the threshold than currently are

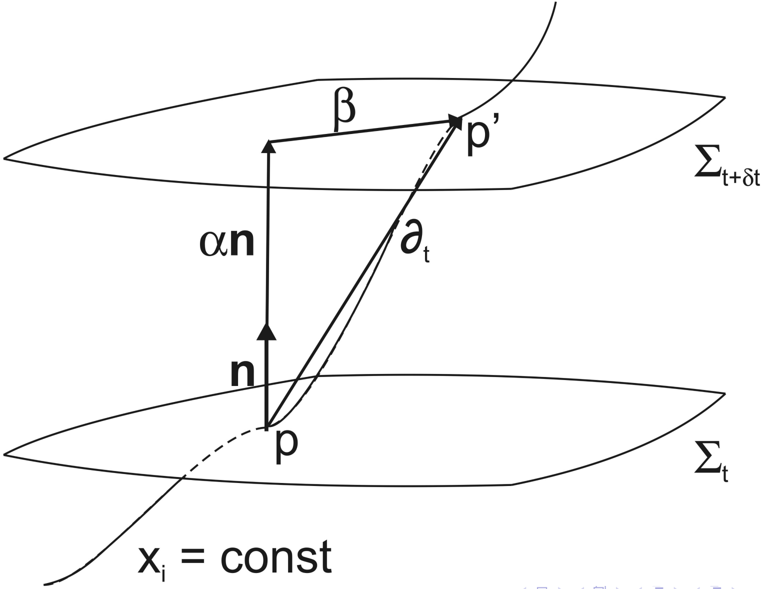
PhD goal

- Understanding the physics of extreme spacetimes
- Improving numerical relativity tools to study these extreme spacetimes
- Deeply study and understand the vacuum spacetimes close to the threshold of black hole formation
- Ultimately, make inroads into possible violations of the cosmic censorship conjecture

Thank you!

BACK UP

(3+1) decomposition



[1] equations

Auxiliar fields

$$\Phi \equiv \phi' \qquad \Pi \equiv \frac{a}{\alpha}\dot{\phi} \tag{4.1}$$

Constraint equations

$$\frac{\alpha'}{\alpha} - \frac{a'}{a} + \frac{1-a^2}{r} = 0 \tag{4.2}$$

$$\frac{a'}{a} + \frac{a^2-1}{2r} - 2\pi r\left(\Pi^2 + \Phi^2\right) = 0 \tag{4.3}$$

Evolution equations

$$\dot{\Phi} = \left(\frac{\alpha}{a}\Pi\right)' \qquad \dot{\Pi} = \frac{1}{r^2}\left(r^2\frac{\alpha}{a}\Phi\right)' \tag{4.4}$$

Axisymmetric spacetime

$$ds^2 = -\alpha^2 dt^2 + \phi^4 [e^{2\eta/3} (dr + \beta^r dt)^2 + r^2 e^{2\eta/3} \left(d\theta + \beta^\theta dt \right)^2 + e^{-4\eta/3} r^2 sin^2(\theta) d\varphi^2]$$