Light ring stability in ultra-compact objects

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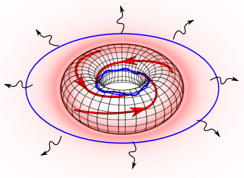

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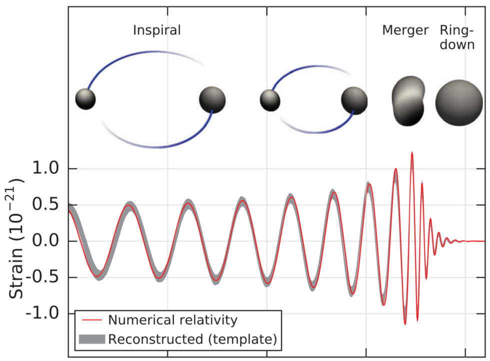
P. Cunha, E. Berti and C. Herdeiro

LIGO events sourced by BH-mimickers?



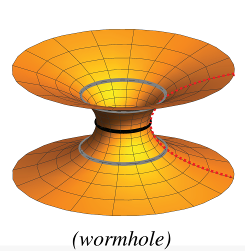
source: LIGO

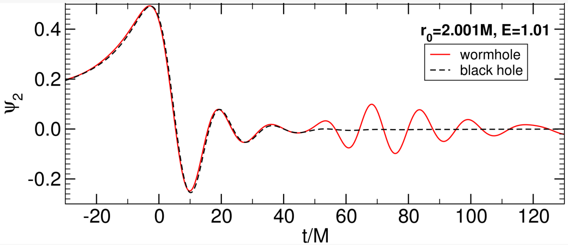
First LIGO event:

- consistent with a Black Hole (BH) merger.
- also compatible with other objects?

Ringdown as an horizon probe?

From Cardoso+2016:

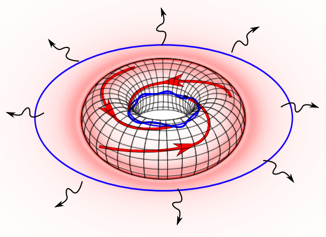




source: PRL**117**, 089902

- LIGO ringdown has signature of a Light Ring (LR).
- Object with a LR $\rightarrow$ Ultra Compact Object (UCO)
- *Horizonless* UCO $\rightarrow$ would also vibrate like a BH (initially).
- Are these objects viable BH-mimickers?

Outline of the argument

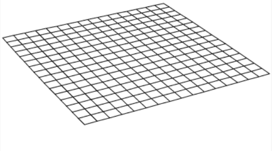


$\Longrightarrow$ horizonless UCOs might be *unstable*, within generic conditions.

- Reasonable assumptions, *e.g.* smoothness, causality and axial symmetry.
- LRs come in pairs $\rightarrow$ one is *stable* (unless NEC is violated).
- Stable LR traps radiation $\rightarrow$ destabilizes object.

Assumptions for the spacetime

$\simeq$ flat spacetime



$\longrightarrow$

UCO

We assume:

- dynamical formation from gravitational collapse.
- initial flat spacetime and causality $\implies$ topological triviality (Geroch).

- UCO is stationary, axially-symmetric and asymptotically flat.
- *no* event horizon; $\mathbb{Z}_2$ reflection symmetry *not* needed.
- The metric is *smooth*.

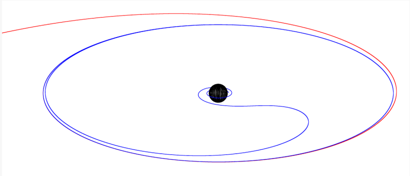
Spacetime metric

We consider a metric:

- 4D, in quasi-isotropic coord. (t, r, θ, φ).
- with Killing vectors ∂_t (stationarity) and ∂_φ (axial-symmetry).

- with $g_{r\theta} = 0, \;\; g_{rr} > 0, \;\; g_{\theta\theta} > 0$ (gauge freedom).
- with $g_{\varphi\varphi} > 0$ (preserve causality).
- until otherwise specified $\rightarrow$ no assumptions on field equations.

Light rings

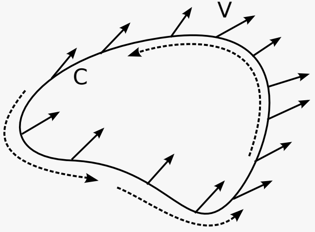


- *Light ring* (LR) $\iff$ circular null geodesic.
- Tangent vector field is a linear combination of (only) $\partial_t, \partial_\varphi$.
- One can define the 2D effective potentials $H_\pm(r,\theta)$ and $U(r,\theta)$.

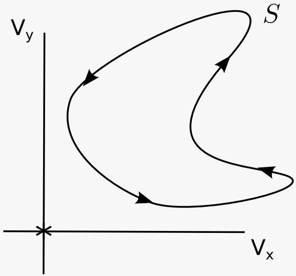
At a LR: $\implies$ $\boxed{\nabla H_\pm = 0}$ (critical point of $H_\pm$)

or: $\nabla U = 0$ and $U = 0$

Winding number

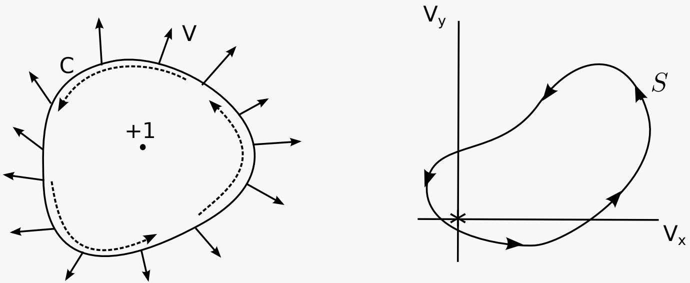


$w = 0$



- Consider a closed 2D contour C with a 2D field $V = \nabla H_{\pm}$.
- The circulation of V around C is mapped to a curve $S(V_x, V_y)$.
- The winding number around $V = 0$ is a topological quantity w.

Winding number



$$w = +1$$

- Consider a closed 2D contour C with a 2D field $V = \nabla H_{\pm}$.
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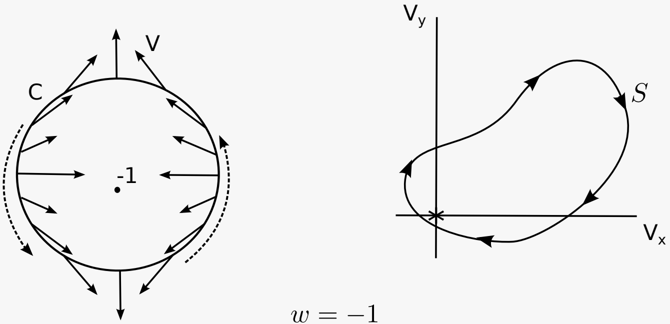
Winding number



$w = -1$

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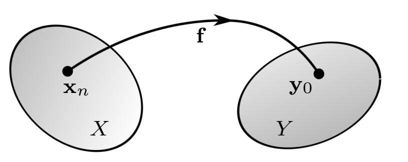
Brouwer degree (topology)



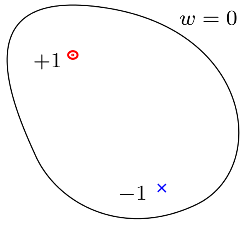
Consider a smooth map $\mathbf{f} : X \to Y$

- take a regular value $\mathbf{y}_0 \in Y$ with finite solutions to $\mathbf{f}(\mathbf{x}_n) = \mathbf{y}_0$.
- the Jacobian $J_n = \det(\partial \mathbf{f} / \partial \mathbf{x}_n) \neq 0$ is computed at each $\mathbf{x}_n$.

The Brouwer degree of **f** is: $w = \sum_n \text{sign}(J_n)$.

- It is independent on the choice $\mathbf{y}_0$.
- It is invariant under homotopies (continuous deformations of the map).

Brouwer degree (topology)



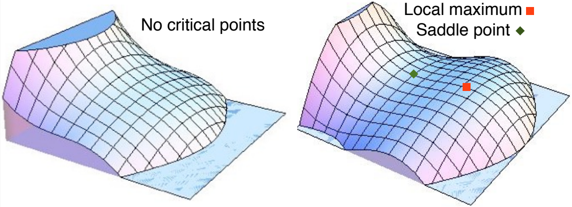
Each critical point $\nabla H_{\pm} = 0$:

- is assigned a *topological* charge w.
- sign w depends on the Jacobian $J_n = |\partial^2 H_{\pm} / \partial^2 \mathbf{x}_n|$.

Charge of a critical point:

- maximum/minimum $\implies$ $w = +1$.
- saddle point $\implies$ $w = -1$.

Illustration Brouwer degree w

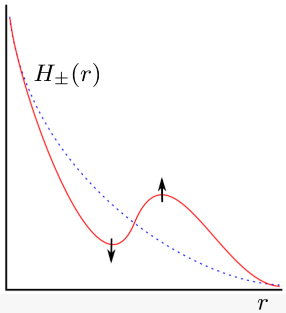


- Illustrative potential $H(x,y) = x(x^2 - a) - (1 + x^2)y^2$.
- Conservation of w under a *smooth* deformation of $(x,y) \to \nabla H$.

- $a = -2 \to$ no critical points $\to w = 0$ (Left).
- $a = 1 \to$ two critical points, $w = +1 - 1 = 0$ (Right).

Simpler case: spherical symmetry

- In spherical symmetry $\rightarrow$ potential $H_{\pm}(r)$ is 1D.

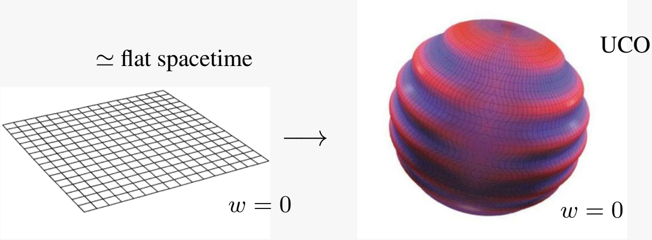


Smooth deformation fixing:

- asymptotic behavior (asymptotic flatness).
- near origin behavior (smoothness).

$\Longrightarrow$ Extrema are created in pairs.

LRs are created in pairs



Flat spacetime (start):

- no LRs $\implies \nabla H_{\pm} \neq 0 \implies$ total $w = 0$.

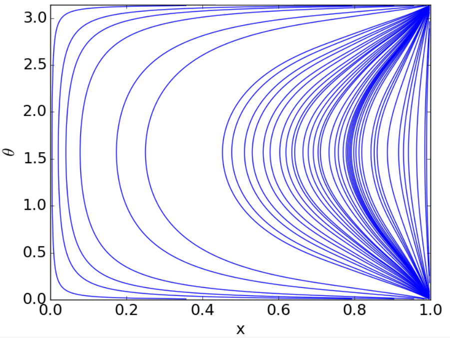
UCO (final):

- UCO can be smoothly deformed into flat spacetime.
- total w still zero.
- LRs must be formed in pairs.

Example: Proca Stars

Contour lines of H_-

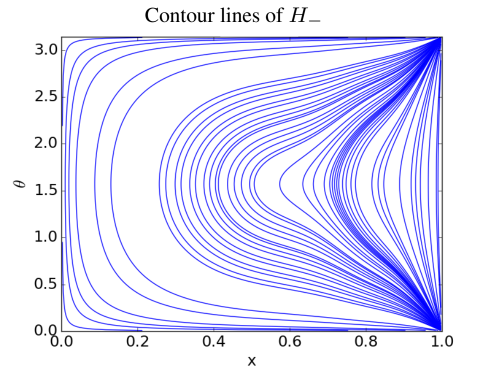
$\simeq$ flat spacetime $\rightarrow$



$x \equiv r/(1+r)$

- Continuous family of spacetimes: *Proca Stars*.
- Sequence of solutions $\rightarrow$ deformation of $H_\pm$.
- Flat spacetime $\neq$ flat $H_\pm$

Example: Proca Stars



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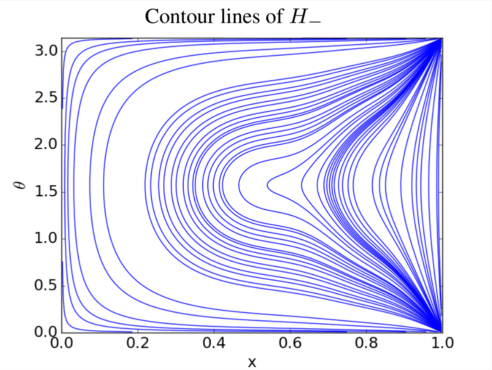

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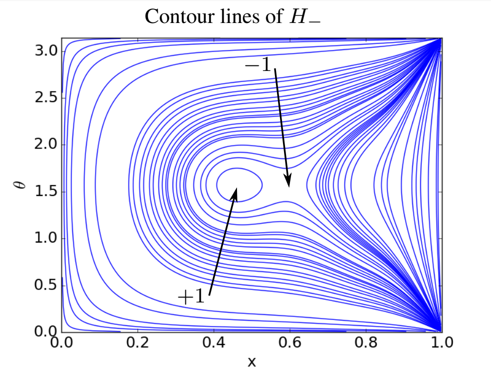

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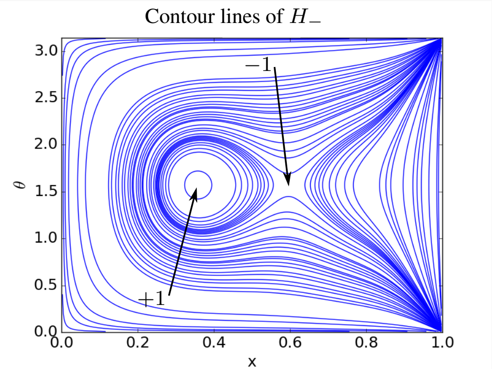

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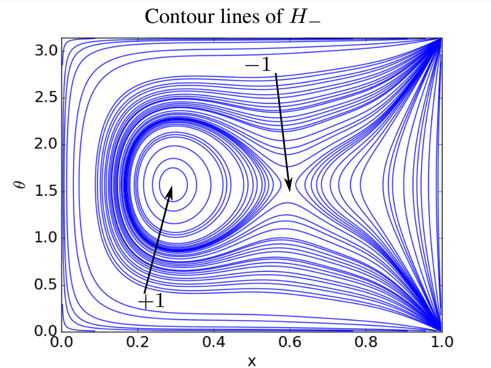

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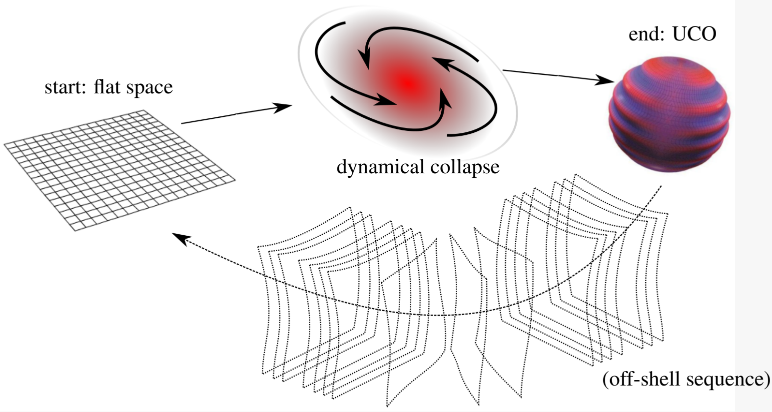
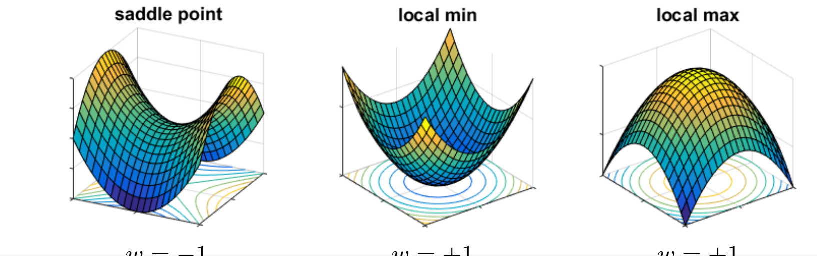
Deformation between endstates



- Endpoints are stationary and axially-symmetric.
- Intermediate collapse stage is not (generically).
- Endpoints can be connected via an off-shell sequence.

Light Ring types



$w = -1$ $\qquad$ $w = +1$ $\qquad$ $w = +1$

Different types of LRs:

- Saddle point of $U \rightarrow$ unstable LR $(w = -1) \rightarrow$ GW signal.
- Local minimum of $U \rightarrow$ stable LR $(w = +1) \rightarrow$ spacetime instability.
- Local maximum of $U \rightarrow$ unstable LR $(w = +1) \rightarrow$ *exotic* LR.

NEC

The *Null Energy Condition* (NEC) states that:

$$T^{\mu\nu}k_\mu k_\nu \geqslant 0, \qquad \forall\, k^\mu : \quad k_\mu k^\mu = 0$$

- The NEC plays a pivotal role in GR.
- It is a critical assumption of Penrose's singularity theorem.
- Robust assumption of a healthy theory of gravity (with exceptions).

Relation to NEC

Consider Einstein's field equations:

$$G^{\mu\nu} = 8\pi\, T^{\mu\nu}.$$

At a LR:

$$\boxed{T^{\mu\nu}\, p_\mu\, p_\nu = \frac{1}{16\pi}\partial_i\partial^i U.}$$

If the LR is exotic (local maximum of U):

- $\partial_i\partial^i U < 0 \implies T^{\mu\nu} p_\mu p_\nu < 0.$
- Null Energy Condition (NEC) is violated for an exotic LR!
- Enforcing NEC $\implies$ UCO has a stable LR.

Further remarks

We further remark:

$$\text{Exotic LR} \implies \text{NEC violation}$$

$$\text{NEC violation} \;\not\Longrightarrow\; \text{Exotic LR}$$

$\rightarrow$ the NEC can be violated at some point other than a LR.

$\rightarrow$ an exotic LR requires an anti-gravity like interaction.

Since $T^{\mu\nu}\,p_\mu\,p_\nu = \partial_i\partial^i U/(16\pi)$:

- stable and exotic LRs are not possible in vaccum.

Conclusions

Under generic and reasonable physical conditions:

- Light Rings are created in pairs.
- If the Null Energy Condition is preserved, one of the LRs is stable.

- BH mimickers are potentially unstable.
- The first observations from LIGO should be really from BHs.

Acknowledgements

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