Charged particle collisions as a way to extract energy from rotating electrovacuum black holes

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4$^{\text{th}}$ IDPASC Students Workshop

IDPASC

TÉCNICO LISBOA

FCT Fundação para a Ciência e a Tecnologia

Background of the topic:

- R. Penrose, *Gravitational Collapse: the Role of General Relativity*, Rivista del Nuovo Cimento, Numero Speziale, 252 (1969).
- Energy extraction by injecting a particle with negative energy
- M. Bañados, J. Silk, S. M. West, *Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy*, PRL **103**, 111102 (2009).
- High-energy, near-horizon collisions of test particles

Many contras:

- Requires fine-tuned particles moving around extremal black holes
- Neglects all sorts of back-reaction, long timescale of the process etc.

Why still study it? And why bother to add charge?

- Geometrical nature, probe of AdS_2-like region of extremal black holes
- More sophisticated processes are possible for subextremal black holes; extremal black hole is a simplified "best-case scenario" of a fast spinning astrophysical black hole
- Black holes can maintain a small charge due to interaction with external magnetic field: R. M. Wald, *Black hole in a uniform magnetic field*, Phys. Rev. D **10**, 1680-1685 (1974).

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Canonical review

- General axially symmetric stationary metric as a model of an isolated black hole, $N \to 0$ at the horizon(s), outer horizon at r_+, extremal r_0

$$\boldsymbol{g} = -N^2 \, \mathbf{d}t^2 + g_{\varphi\varphi} \left(\mathbf{d}\varphi - \omega \, \mathbf{d}t\right)^2 + g_{rr} \, \mathbf{d}r^2 + g_{\vartheta\vartheta} \, \mathbf{d}\vartheta^2$$

- Electromagnetic potential and generalised electrostatic potential ϕ

$$\boldsymbol{A} = A_t \, \mathbf{d}t + A_\varphi \, \mathbf{d}\varphi = -\phi \, \mathbf{d}t + A_\varphi \left(\mathbf{d}\varphi - \omega \, \mathbf{d}t\right)$$

- Canonical formalism (analogous to classical mechanics) for motion of a test particle with charge $q = m\tilde{q}$

$$\mathscr{L} = \frac{1}{2} m g_{\mu\nu} u^\mu u^\nu + q A_\mu u^\mu \qquad \mathit{\Pi}_\alpha = \frac{\partial \mathscr{L}}{\partial u^\alpha} = p_\alpha + q A_\alpha$$

- Constants of motion (E, L_z) and reduced (per unit rest mass) constants of motion (ε, l) of a particle with rest mass m

$$-\mathit{\Pi}_t = -p_t - qA_t = E = \varepsilon m$$

$$\mathit{\Pi}_\varphi = p_\varphi + qA_\varphi = L_z = lm$$

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First-order equations of motion

- Two constants of motion give two first-order equations of motion

$$p^t = \frac{\mathcal{X}}{N^2} \qquad p^\varphi = \frac{\omega \mathcal{X}}{N^2} + \frac{L_z - qA_\varphi}{g_{\varphi\varphi}}$$

- "Forwardness"

$$\mathcal{X} \equiv E - \omega L_z - q\phi > 0$$

- Assuming "mirror symmetry" and initial conditions $\vartheta = \pi/2$, $u^\vartheta = 0$, we can get the third equation from velocity normalisation,

$$p^r = \sigma \sqrt{\frac{1}{N^2 g_{rr}} \left[\mathcal{X}^2 - N^2 \left(m^2 + \frac{(L_z - qA_\varphi)^2}{g_{\varphi\varphi}} \right) \right]}$$

- Factorizing the expression for massive particles, we get an effective potential ($\varepsilon \geqslant V$, analogy of a classical potential), photons different

$$V = \omega l + \tilde{q}\phi + N\sqrt{1 + \frac{(l - \tilde{q}A_\varphi)^2}{g_{\varphi\varphi}}}$$

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Critical particles and near-horizon expansions

- Due to the causality restriction ($p^t > 0$), we can say that particles with $\mathcal{X}_{\mathrm{H}} > 0$ fall into the black hole, whereas particles with $\mathcal{X}_{\mathrm{H}} < 0$ can not get close to horizon
- Fine-tuned particles with $\mathcal{X}_{\mathrm{H}} = 0$ are on the verge between those cases, so they are called "critical particles"
- To describe motion near the horizon ($r = r_0$), we can use expansions; abbreviations for expansion coefficients

$$\tilde{\omega} = \left.\frac{\partial \omega}{\partial r}\right|_{r=r_0} \qquad \tilde{\phi} = \left.\frac{\partial \phi}{\partial r}\right|_{r=r_0} \qquad \tilde{N}_{\mathrm{H}}^2 = \frac{1}{2}\left.\frac{\partial^2 N^2}{\partial r^2}\right|_{r=r_0}$$

- Instead of parameters E, L, q, we can use $\mathcal{X}_{\mathrm{H}}, x, \lambda$

$$x \equiv \left.\frac{\partial \mathcal{X}}{\partial r}\right|_{r=r_0} = -\tilde{\omega}L - q\tilde{\phi} \qquad \lambda \equiv p_{\varphi}^{\mathrm{H}} = L - qA_{\varphi}^{\mathrm{H}}$$

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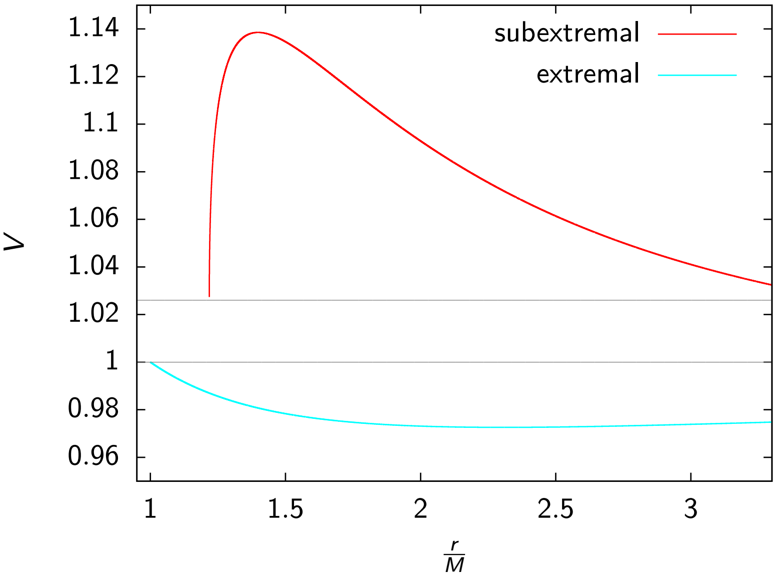
Curves of the effective potential for Kerr-Newman black holes



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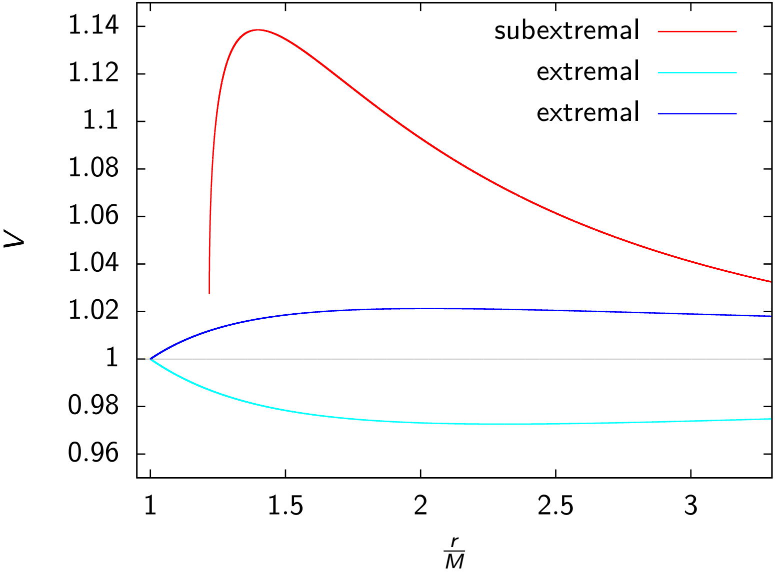

Approach kinematics and the hyperbola

- $\mathcal{X}_{\mathrm{H}} = 0$ implies $\varepsilon = V|_{r_+}$, the effective potential must decrease in order for the motion of critical particles towards r_+ to be allowed
- In the subextremal case the derivative $\partial V/\partial r|_{r_+}$ is always infinite and positive, so no critical particle can approach r_+
- For extremal black holes the derivative is finite, namely

$$m \left.\frac{\partial V}{\partial r}\right|_{r=r_0} = -x + \tilde{N}_{\mathrm{H}} \sqrt{m^2 + \frac{\lambda^2}{g_{\varphi\varphi}^{\mathrm{H}}}}$$

- The "border" of an "admissible region" in the parameter space is defined by the condition

$$\left.\frac{\partial V}{\partial r}\right|_{r=r_0} = 0 \quad \Longleftrightarrow \quad x = \tilde{N}_{\mathrm{H}} \sqrt{m^2 + \frac{\lambda^2}{g_{\varphi\varphi}^{\mathrm{H}}}}$$

- It is a branch of a hyperbola in variables x, λ, or in L, q, if we invert the change of variables (see F.H., J. Bičák, arXiv:1612.04959, PhysRevD.95.084055 for details of approach kinematics)

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Nearly-critical particles

- If the motion of a critical particle is allowed, it has the character of exponential relaxation towards r_0 with higher order corrections

$$r \doteq r_0 \left[1 + \exp\left(-\frac{\tau}{\tau_{\text{relax}}}\right)\right] + \dots$$

- No critical particle will ever reach r_0, but collision with another particle can occur arbitrarily close to r_0, at some radius r_C
- If a particle has $\mathcal{X}_H \neq 0$, yet $\mathcal{X}_H \sim (r_C - r_0)$, it will effectively behave as critical in the collision
- To account for this, we define the parameter C with the conventional minus sign (in fact, we need a series expansion for consistency of higher orders of momentum conservation)

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Conservation of momentum

- Let us consider a $2 \to 2$ scattering process involving a critical particle 1 and a usual particle 2, both moving towards extremal black hole
- E_{CM} attainable in such processes diverges $\sim (r_{\mathrm{C}} - r_0)$
- Can we produce highly energetic particles and extract energy?
- Conservation of charge, energy and angular momentum
- For radial momentum, we can do a resummation. For usual particles:

$$N^2 p^t - \sigma N \sqrt{g_{rr}} p^r \sim (r_{\mathrm{C}} - r_0)^2$$
$$N^2 p^t + \sigma N \sqrt{g_{rr}} p^r \doteq 2\mathcal{X}_{\mathrm{H}} + \ldots$$

- In contrast, for (nearly-)critical particles

$$N^2 p^t \pm N \sqrt{g_{rr}} p^r \sim (r_{\mathrm{C}} - r_0)$$

- Thus, we can split orders by summing the time and radial component

$$N^2 \left(p^t_{(1)} + p^t_{(2)} \right) + N\sqrt{g_{rr}} \left(p^r_{(1)} + p^r_{(2)} \right) = N^2 \left(p^t_{(3)} + p^t_{(4)} \right) + N\sqrt{g_{rr}} \left(p^r_{(3)} + p^r_{(4)} \right)$$

- One of the produced particles (No. 4) must fall into the black hole

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Conservation of momentum II

- Particle 3 must be (nearly-)critical and its fate is determined by the leading order of the previous equation

$$x_1 - \sqrt{x_1^2 - \tilde{N}_{\mathrm{H}}^2 \left(m_1^2 + \frac{\lambda_1^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right)} =$$

$$= x_3 - C_3 + \sigma_3 \sqrt{(x_3 - C_3)^2 - \tilde{N}_{\mathrm{H}}^2 \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right)}$$

- We can solve for C_3 and for σ_3

$$C_3 = x_3 - \frac{1}{2} \left[\mathfrak{A}_1 + \frac{\tilde{N}_{\mathrm{H}}^2}{\mathfrak{A}_1} \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right) \right]$$

$$\sigma_3 = \operatorname{sgn} \left[\mathfrak{A}_1^2 - \tilde{N}_{\mathrm{H}}^2 \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right) \right]$$

- Four kinematic regimes are possible based on sign of C_3 and on σ_3

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$$= x_3 - C_3 + \sigma_3 \sqrt{(x_3 - C_3)^2 - \tilde{N}_{\mathrm{H}}^2 \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right)}$$

- We can solve for C_3 and for σ_3

$$C_3 = x_3 - \frac{1}{2} \left[\mathfrak{A}_1 + \frac{\tilde{N}_{\mathrm{H}}^2}{\mathfrak{A}_1} \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right) \right]$$

$$\sigma_3 = \operatorname{sgn} \left[\mathfrak{A}_1^2 - \tilde{N}_{\mathrm{H}}^2 \left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}} \right) \right]$$

- Four kinematic regimes are possible based on sign of C_3 and on σ_3

Fate of particle 3

- If $C_3 > 0$ ("+" regimes), particle 3 will have $\mathcal{X}_{\mathrm{H}} < 0$ and can not fall into the black hole, even if it is initially incoming, it must be reflected by the effective potential
- For $C_3 < 0$ and $\sigma_3 = -1$ (IN$-$ regime) particle 3 will certainly fall into the black hole and can not extract energy
- Regimes with $\sigma_3 = +1$ ("OUT") are possible only when $\mathfrak{A}_1 > \tilde{N}_{\mathrm{H}} m_3$
- The four kinematic regimes can be represented as regions in the parameter space x, λ or L, q
- Condition $C_3 = 0$ is a parabola in the parameter space

$$x_3 = \frac{1}{2}\left[\mathfrak{A}_1 + \frac{\tilde{N}_{\mathrm{H}}^2}{\mathfrak{A}_1}\left(m_3^2 + \frac{\lambda_3^2}{g_{\varphi\varphi}^{\mathrm{H}}}\right)\right]$$

- Condition "$\sigma_3 = 0$" (when the expansion breaks down) is a pair of straight lines

$$\lambda_3 = \pm\sqrt{g_{\varphi\varphi}^{\mathrm{H}}\left(\frac{\mathfrak{A}_1^2}{\tilde{N}_{\mathrm{H}}^2} - m_3^2\right)}$$

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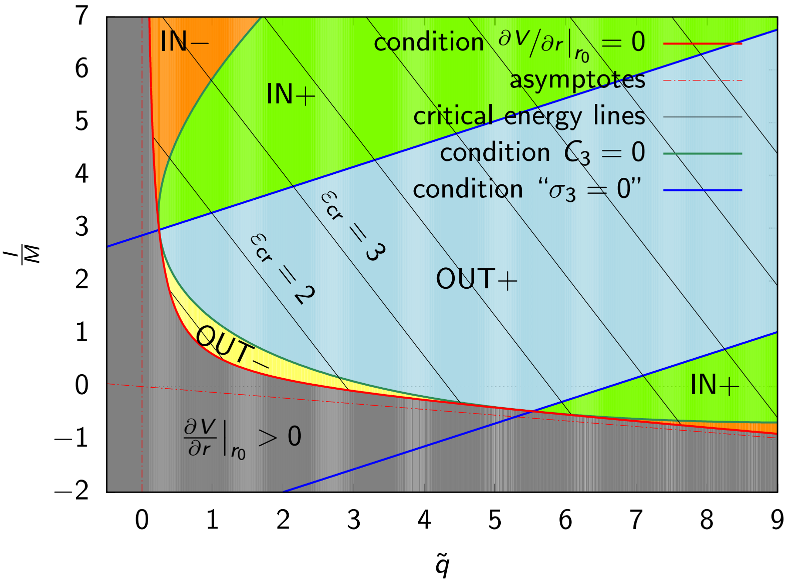
Extremal Kerr-Newman solution with $\frac{a}{M}=\frac{1}{2},\ \frac{Q}{M}=\frac{\sqrt{3}}{2}$



Conclusions

- For uncharged particles, there is an upper bound on extracted energy even without back-reaction: T. Harada, H. Nemoto, U. Miyamoto, *Upper limits of particle emission from high-energy collision and reaction near a maximally rotating Kerr black hole*, Phys. Rev. D **86**, 024027 (2012).
- This upper bound disappears when we include charge
- However, a lot of things may go wrong in a more realistic treatment: charge quantisation/quasineutrality, consistency of expansions, bremsstrahlung... a lot of further work is possible

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- The work was supported by FCT (IDPASC) grant Bolsa de Investigação, reference PD/BD/113477/2015