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Influence of intrinsic spin in the formation of singularities for inhomogeneous dust space-times

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Introduction

How to account for spin effects?

The Einstein-Cartan theory allows a non-vanishing space-time torsion field that can be related with the matter (intrinsic) spin.

Einstein-Cartan theory is not GR, is it?

Given a fluid with zero vorticity and composed of microscopic particles with randomly oriented spin, computing the average energy-momentum tensor over a volume, the field equations read

$$\widetilde{G}_{\mu\nu} = 8\pi \left[\left(\rho + p - 4\pi s^2\right) u_\mu u_\nu + \left(p - 2\pi s^2\right) g_{\mu\nu} \right]$$

where $\tilde{G}_{\mu\nu}$ is the Einstein tensor computed only from the metric connection, s^2 the average of the square of the spin density, ρ and p represent the mass-energy density and the pressure of the fluid.

Introduction

Re-writing the previous equation as

$$G_{\mu\nu} = 8\pi \left[(\rho_{\text{eff}} + p_{\text{eff}})\, u_\mu u_\nu + g_{\mu\nu}\, p_{\text{eff}} \right]$$

where

$$\begin{cases} \rho_{\text{eff}} = \rho - 2\pi\, s^2 \\ p_{\text{eff}} = p - 2\pi\, s^2 \end{cases}$$

allow us to conclude that the previous setup can be effectively described by GR, assuming that the fluid is described by the effective energy-momentum tensor

$$(T_{\text{eff}})_{\mu\nu} = (\rho_{\text{eff}} + p_{\text{eff}})\, u_\mu u_\nu + g_{\mu\nu}\, p_{\text{eff}}$$

Introduction

Was the influence of spin in singularity formation studied in the past?

In Nature Phys. Sci. **242**, 7 (1973), it was proved that spin effects do prevent a cosmological singularity in a FLRW space-time. However, in Nature Phys. Sci. **244**, 96 (1973) and in Phys. Lett. A **43A**, 63 (1973), was shown that if the space-time is not isotropic, a singularity may not be prevented.

More articles came out more recently, however, previous studies assume some of the following

- Isotropic or homogenous models;
- Spins are aligned in a given preferred direction;
- The fluid obeys an EoS in the form: $p = k\rho$ with $k < 1$.

Setup

- The space-time is permeated by a fluid composed only of uncharged fermions.
- The fluid effectively behaves as irrotational effective dust: $p_{eff} = 0$, such that

$$(T_{\rm eff})_{\mu\nu} = \rho_{\rm eff}\, u_\mu u_\nu$$

- Within the collapsing matter cloud the space-time is described by a Szekeres solution.

Szekeres space-times

The Szekeres space-times represents all solutions of the EFE for a space-time permeated by irrotational dust whose line element can be written in the form

$$ds^2 = -d\tau^2 + e^{\alpha(\tau,r,p,q)}dr^2 + e^{\beta(\tau,r,p,q)}\left(dp^2 + dq^2\right)$$

In general the Szekeres solutions have no Killing vector fields, that is, in general such space-times have isometries.

Spin effects in the evolution of the collapse

Historically, the Szekeres solutions were separated in two classes however, it has been shown that there is in fact only one type of Szekeres space-times, which are characterized by the line element

$$ds^2 = -d\tau^2 + \frac{\left(R' - \frac{R\,E'}{E}\right)^2}{\epsilon + f(r)} dr^2 + \frac{R^2}{E^2}\left(dp^2 + dq^2\right) \ ,$$

where $R \equiv R(\tau, r)$, $E \equiv E(r, p, q)$, $\epsilon = \{-1, 0, 1\}$, $f(r) + \epsilon > 0$ and $' \equiv \partial_r$.

Moreover, it is useful to define

$$W(\tau, r, p, q) = E(r, p, q)R'(\tau, r) - R(\tau, r)E'(r, p, q)$$

Spin effects in the evolution of the collapse

Initial regularity conditions

Assumption 1. *At the initial instant,* τ_0 *, the function* $R(\tau_0, r)$ *is a monotonically increasing function of the coordinate* r *.*

Assumption 2. *At the initial instant, the space-time is regular, in the sense that a possible curvature singularity will only form later during the collapse.*

Assumption 3. *At the initial instant,*

$$\mathrm{sign}(\rho_{eff}(\tau_0, r, p, q)) = \mathrm{sign}(W(\tau_0, r, p, q)),$$

at every point within the matter cloud.

Spin effects in the evolution of the collapse

Theorem. *Given a Szekeres space-time permeated by an uncharged collapsing perfect fluid, composed only of fermionic particles, characterized by an equation of state such that, the fluid effectively behaves as dust, if assumptions 1-3 are verified, a curvature singularity will not form.*

Special Cases

- Lemaître-Tolman-Bondi space-times
- Friedman-Lemaître-Robertson-Walker space-times
- Senovilla-Vera space-times
- Locally rotationally symmetric Bianchi type I space-times
- Kantowski-Sachs space-times

Roadmap

The fluid is composed only of uncharged fermions

The fluid effectively behaves as irrotational effective dust

The space-time is spherically symmetric

The collapsing fluid is marginally bound

$\tau + \delta\tau$ →

The fluid is composed only of uncharged fermions.

The fluid effectively behaves as irrotational effective dust.

The space-time is spherically symmetric.

Roadmap

The fluid is composed only of uncharged fermions.

The fluid effectively behaves as irrotational effective dust.

The space-time is spherically symmetric.

$\tau + \Delta\tau$ →

The fluid is composed only of uncharged fermions.

The fluid effectively behaves as irrotational effective dust.

The space-time is described by a Szekeres solution

Roadmap

The fluid is composed only of uncharged fermions

The fluid effectively behaves as irrotational effective dust

The space-time is described by a Szekeres solution

→

The fluid is composed of fermions and bosons

The fluid obeys a wide range of equations of state

The space-time is described by a general solution of the EFE