Beyond the Standard Model
Neutrinos, Muon g-2 and Extra Matter

Luís M. Cebola
luismcebola@tecnico.ulisboa.pt

CFTP - IST - U. Lisbon

June 29, 2018

Outline

Neutrino Oscillations

The fact that neutrinos oscillate imply non-vanishing masses and non-vanishing mixing angles.

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{2E}L\right)$$

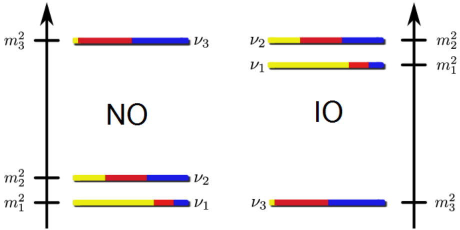
Like in the quarks case with the CKM matrix, for neutrinos, PMNS matrix represents the mismatch between mass states and interaction states that take part in the charged weak current.

$$\nu_i = \sum_j \mathbf{U}^{ij}_{PMNS}\,\nu_j,\;\; i = e, \nu, \tau,\;\; j = 1, 2, 3.$$

Neutrino Oscillations

The absolute scale of neutrino masses is not known and there are two possible orderings of the light neutrino masses: normal ordering (NO) with $m_1 < m_2 < m_3$ or inverted ordering (IO) with $m_3 < m_1 < m_2$.

Possibles orderings:



Neutrino Oscillations

$0\nu\beta\beta$ experiments are yet to confirm the nature of neutrino: Dirac or Majorana particle.

Dirac: $\mathbf{m}_\nu^{\alpha\beta}\,\overline{\nu_{L\alpha}}\,\nu_{R\beta} \rightarrow \mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu = \mathbf{d}_n = \mathrm{diag}\left(m_1, m_2, m_3\right)$
Majorana: $\mathbf{m}_\nu^{\alpha\beta}\,\overline{\nu_{L\alpha}}\,\nu_{L\beta}^c \rightarrow \mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu^* = \mathbf{d}_n = \mathrm{diag}\left(m_1, m_2, m_3\right)$

In the basis where charged lepton are diagonal ($\mathbf{m}_e = \mathbf{d}_e$) we have $\mathbf{U}_{PMNS} = \mathbf{U}_e^\dagger \mathbf{U}_\nu = \mathbf{U}_\nu$

PMNS Matrix

It can be parametrized (Standard parametrization) as

$$\mathbf{U} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times$$

$\mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$,

where c_{ij} and s_{ij} are $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively. The Dirac CP-violating phase is δ and the Majorana phases α_{21} and α_{31} are present only in the Majorana case.

Neutrino Oscillation Data

Parameter	Best fit $\pm$ 1σ	3σ
Δm^2_{21} $[10^{-5}\,\mathrm{eV}^2]$	$7.60^{+0.19}_{-0.18}$	7.11 – 8.18
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\,\mathrm{eV}^2]$ (NO)	$2.48^{+0.05}_{-0.07}$	2.30 – 2.65
(IO)	$2.38^{+0.05}_{-0.06}$	2.20 – 2.54
$\sin^2\theta_{12}/10^{-1}$	3.23 ± 0.16	2.78 – 3.75
$\sin^2\theta_{23}/10^{-1}$ (NO)	$5.67^{+0.32}_{-1.24}$	3.93 – 6.43
(IO)	$5.73^{+0.25}_{-0.39}$	4.03 – 6.40
$\sin^2\theta_{13}/10^{-2}$ (NO)	2.26 ± 0.12	1.90 – 2.62
(IO)	2.29 ± 0.12	1.93 – 2.65
δ/π (NO)	$1.41^{+0.55}_{-0.40}$	0.0 – 2.0
(IO)	1.48 ± 0.31	0.0 – 2.0

Additional Constraints

Two additional constraints from experiments:

- The effective neutrino mass parameter, $m_{\beta\beta} = |\sum_i \mathbf{U}_{ei}^2 m_i|$, as it is related to the $\mathbf{U}$ data.
- The bound on neutrino mass sum, $\sum_i m_i < 0.23$ eV (95 % CL) obtained by the Planck mission, assuming three species of degenerate massive neutrinos and a ΛCDM model.

Flavour Puzzle: How can the data constrain the structure of the neutrino mass matrix ($\mathbf{m}_\nu$)?

The Flavour Puzzle

There is no compelling theory to explain the origin of the lepton flavour structure. In the SM all mass matrices are free.

From the theoretical point of view, a natural approach is to restrict the number of free parameters in the lepton flavour sector so that the theory becomes more predictive.

Usual frameworks for the Majorana neutrino mass matrix ($\mathbf{m}_\nu$):

- Zero textures
- Hybrid textures

FGM Matrices

$$\mathbf{A}_1 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \mathbf{A}_2 : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \mathbf{B}_1 : \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$\mathbf{B}_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \mathbf{B}_3 : \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \mathbf{B}_4 : \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$\mathbf{C} : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \mathbf{D}_1 : \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \mathbf{D}_2 : \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}.$$

Results for FGM Textures

The seven matrices were previously found to be compatible with neutrino oscillation data at the 1σ level for NO and IO mass spectrum.

Majorana $\mathbf{m}_\nu$	χ^2_{min} NO	(IO)	Δm^2_{21}	Δm^2_{31}	θ_{12}	θ_{23}	θ_{13}	δ
$\mathbf{A}_1$	2.92×10^{-1}	(3.81×10^{2})	✓	✓	✓(×)	✓(×)	✓	✓(×)
$\mathbf{A}_2$	1.23×10^{-2}	(3.14×10^{2})	✓	✓	✓(×)	✓(×)	✓	✓(×)
$\mathbf{B}_1$	8.39×10^{-1}	(4.04×10^{-3})	✓	✓	✓	✓	✓	✓
$\mathbf{B}_2$	3.39×10^{-2}	(1.02×10^{1})	✓	✓	✓	✓(×)	✓	✓
$\mathbf{B}_3$	9.12×10^{-1}	(3.45×10^{-3})	✓	✓	✓	✓	✓	✓
$\mathbf{B}_4$	2.10×10^{-2}	(1.11×10^{1})	✓	✓	✓	✓(×)	✓	✓
$\mathbf{C}$	6.20×10^{2}	(1.04×10^{-1})	✓	×(✓)	✓	✓	×(✓)	✓

Textures Results

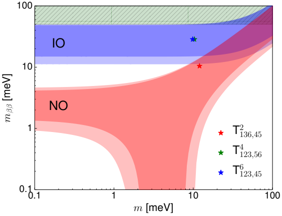
$$T^2_{136,45} : \begin{pmatrix} a & 0 & a \\ 0 & b & b \\ a & b & a \end{pmatrix}$$

$$\mathbf{A}_1 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

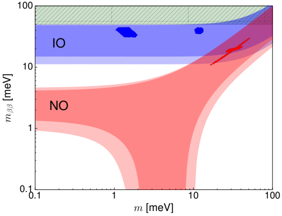
$$T^2_{34} : \begin{pmatrix} * & 0 & a \\ 0 & a & * \\ a & * & * \end{pmatrix}$$

n	Texture class	No. of textures	Solutions $(1\sigma, 3\sigma)$ NO	IO
2	$4_1 0_2$	7^* $(\subset 15)$	-	-
3	$2_2 0_2$	21^* $(\subset 45)$	-	-
	$3_1 1_1 0_2$	28^* $(\subset 60)$	-	-
	$\mathbf{3_1 2_1 0_1}$	60	(0,1)	(0,2)
	3_2	10	-	-
	$4_1 1_1 0_1$	30	-	-
	$4_1 2_1$	15	-	-
4	$2_1 1_2 0_2$	42^* $(\subset 90)$	(4,2)	(1,3)
	$2_2 1_1 0_1$	90	(15,3)	(19,11)
	$3_1 1_2 0_1$	60	(9,3)	(13,10)
	$3_1 2_1 1_1$	60	(13,5)	(13,5)
	$4_1 1_2$	15	(5,0)	(3,1)
5	$\mathbf{1_4 0_2}$	7^* $(\subset 15)$	(6,0)	(3,2)
	$\mathbf{2_1 1_3 0_1}$	60	(31,4)	(36,7)
	2_3	15	(4,0)	(1,1)
	$3_1 1_3$	20	(19,0)	(18,0)

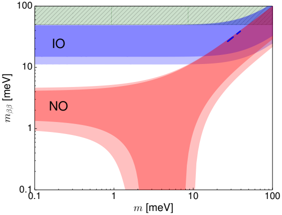
Effective Neutrino Mass Parameter Predictions



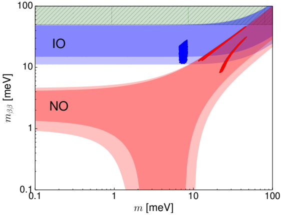
(a) Class $3_1 2_1 0_1$



(b) Class $4_1 1_2$



(c) Class $2_1 1_2 0_2$



(d) Class 2_3

Model and Motivation

Abelian extension of the SM based on an extra $U(1)_X$ gauge symmetry, with $X \equiv a\,B - \sum_\alpha b_\alpha L_\alpha$ being an arbitrary linear combination of the baryon number B and the individual lepton numbers L_α.

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$$

Find all the anomaly-free $U(1)_X$ gauge symmetries that lead to predictive two-zero textures in the effective neutrino mass matrix, obtained via type I seesaw mechanism, with a minimal extra matter content: singlet right-handed neutrinos ν_R and a complex singlet scalar S.

Anomaly Constraints

$$A_1 = n_G \left(2\,x_q - x_u - x_d\right) = 0\,,$$

$$A_2 = \frac{3n_G}{2} x_q + \frac{1}{2}\sum_{i=1}^{n_G} x_{\ell i} = 0\,,$$

$$A_3 = n_G \left(\frac{x_q}{6} - \frac{4x_u}{3} - \frac{x_d}{3}\right) + \sum_{i=1}^{n_G}\left(\frac{x_{\ell i}}{2} - x_{ei}\right) = 0\,,$$

$$A_4 = n_G \left(x_q^2 - 2x_u^2 + x_d^2\right) + \sum_{i=1}^{n_G}\left(-x_{\ell i}^2 + x_{ei}^2\right) = 0\,,$$

$$A_5 = n_G \left(6\,x_q^3 - 3\,x_u^3 - 3\,x_d^3\right) + \sum_{i=1}^{n_G}\left(2\,x_{\ell i}^3 - x_{ei}^3\right) - \sum_{i=1}^{n_R} x_{\nu i}^3 = 0\,,$$

$$A_6 = n_G \left(6x_q - 3x_u - 3x_d\right) + \sum_{i=1}^{n_G}\left(2\,x_{\ell i} - x_{ei}\right) - \sum_{i=1}^{n_R} x_{\nu i} = 0\,.$$

Fermion Masses

The Lagrangian terms that generate the SM fermion masses and are compatible with type I seesaw models for Majorana neutrinos are given by

$$\mathbf{Y}_u\,\overline{q_L}u_R\widetilde{H}+\mathbf{Y}_d\,\overline{q_L}d_RH+\mathbf{Y}_e\,\overline{\ell_L}e_RH+\mathbf{Y}_\nu\,\overline{\ell_L}\nu_R\widetilde{H}$$
$$+\frac{1}{2}\mathbf{m}_R\nu_R^TC\nu_R+\text{H.c.},$$

Fermion Masses and Seesaw Mechanism

Including a complex singlet scalar S, the Lagrangian terms that generate the SM fermion masses and are compatible with type I seesaw models for Majorana neutrinos are given by

$$\mathbf{Y}_u\,\overline{q_L}u_R\widetilde{H} + \mathbf{Y}_d\,\overline{q_L}d_R H + \mathbf{Y}_e\,\overline{\ell_L}e_R H + \mathbf{Y}_\nu\,\overline{\ell_L}\nu_R\widetilde{H}$$
$$+\frac{1}{2}\mathbf{m}_R\nu_R^T C\nu_R + \mathbf{Y}_1\,\nu_R^T C\nu_R S + \mathbf{Y}_2\,\nu_R^T C\nu_R S^* + \text{H.c.},$$

The effective neutrino mass matrix generated by the seesaw mechanism is

$$\mathbf{m}_\nu \simeq -\mathbf{m}_D\;\mathbf{M}_R^{-1}\;\mathbf{m}_D^T\,,$$

where

$$\mathbf{m}_D = \mathbf{Y}_\nu\langle H\rangle, \quad \mathbf{M}_R = \mathbf{m}_R + 2\mathbf{Y}_1\langle S\rangle + 2\mathbf{Y}_2\langle S^*\rangle.$$

FGM Matrices

If $\mathbf{m}_\nu \simeq -\mathbf{m}_D \ \mathbf{M}_R^{-1} \ \mathbf{m}_D^T$ is a FGM matrix it (should) work.

$$\mathbf{A}_1 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \mathbf{A}_2 : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \mathbf{B}_1 : \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$\mathbf{B}_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \mathbf{B}_3 : \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \mathbf{B}_4 : \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$\mathbf{C} : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \mathbf{D}_1 : \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \mathbf{D}_2 : \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}.$$

Results

Diagonal Dirac-neutrino mass matrix $\mathbf{m}_D$:

Symmetry generator X	$\lvert x_s \rvert$	$\mathbf{M}_R$	$\mathbf{m}_\nu$
$B + L_e - L_\mu - 3L_\tau$	2	$\mathbf{D}_2$	$\mathbf{A}_1$
$B + 3L_e - L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{23} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_e - 6L_\tau$	3		
$B + 9L_e - 3L_\mu - 9L_\tau$	6		
$B + L_e - 3L_\mu - L_\tau$	2	$\mathbf{D}_1$	$\mathbf{A}_2$
$B + 3L_e - 5L_\mu - L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{23} = 0$	
$B + 3L_e - 6L_\mu$	3		
$B + 9L_e - 9L_\mu - 3L_\tau$	6		
$B - L_e + L_\mu - 3L_\tau$	2	$\mathbf{B}_4$	$\mathbf{B}_3$
$B - L_e + 3L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{13} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_\mu - 6L_\tau$	3		
$B - 3L_e + 9L_\mu - 9L_\tau$	6		
$B - L_e - 3L_\mu + L_\tau$	2	$\mathbf{B}_3$	$\mathbf{B}_4$
$B - L_e - 5L_\mu + 3L_\tau$	2	$(\mathbf{M}_R)_{12} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B - 6L_\mu + 3L_\tau$	3		
$B - 3L_e - 9L_\mu + 9L_\tau$	6		

Results with Experimental Data

Diagonal Dirac-neutrino mass matrix $\mathbf{m}_D$:

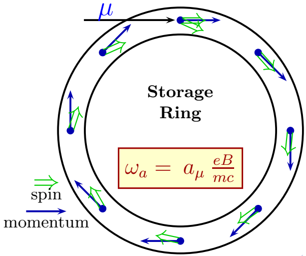
Symmetry generator X	$\lvert x_s \rvert$	$\mathbf{M}_R$	$\mathbf{m}_\nu$
$\mathbf{B + L_e - L_\mu - 3L_\tau}$	$\mathbf{2}$	$\mathbf{D}_2$	$\mathbf{A}_1$
$B + 3L_e - L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{23} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_e - 6L_\tau$	3		
$B + 9L_e - 3L_\mu - 9L_\tau$	6		
$\mathbf{B + L_e - 3L_\mu - L_\tau}$	$\mathbf{2}$	$\mathbf{D}_1$	$\mathbf{A}_2$
$B + 3L_e - 5L_\mu - L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{23} = 0$	
$B + 3L_e - 6L_\mu$	3		
$B + 9L_e - 9L_\mu - 3L_\tau$	6		
$\mathbf{B - L_e + L_\mu - 3L_\tau}$	$\mathbf{2}$	$\mathbf{B}_4$	$\mathbf{B}_3$
$B - L_e + 3L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{13} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_\mu - 6L_\tau$	3		
$B - 3L_e + 9L_\mu - 9L_\tau$	6		
$\mathbf{B - L_e - 3L_\mu + L_\tau}$	$\mathbf{2}$	$\mathbf{B}_3$	$\mathbf{B}_4$
$B - L_e - 5L_\mu + 3L_\tau$	2	$(\mathbf{M}_R)_{12} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B - 6L_\mu + 3L_\tau$	3		
$B - 3L_e - 9L_\mu + 9L_\tau$	6		

Introduction

A particle with spin $\vec{s}$ like the muon exhibits a magnetic moment $\vec{\mu}$:

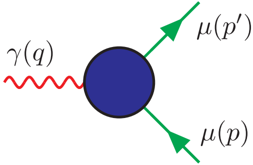
$$\vec{\mu} = g_\mu \frac{e}{2m_\mu}\vec{s} \;\; ; \quad g_\mu = 2\left(1 + a_\mu\right) \;\; ; \quad a_\mu = \frac{g_\mu - 2}{2}$$

Its Dirac value $g_\mu = 2$ is modified by radiative corrections known as the muon anomaly (a_μ) or muon $g-2$.



Computation

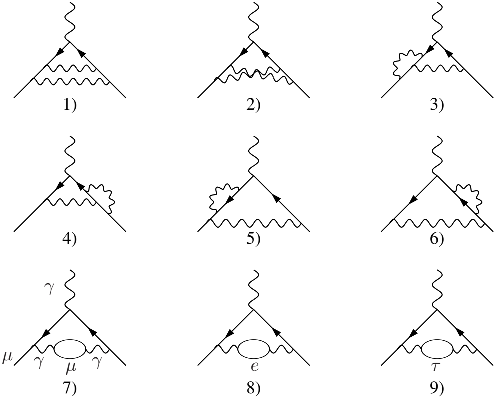
It is tested and calculated in the static limit of the electromagnetic lepton vertex ($q \to 0$).



$$= (-ie)\bar{u}(p')\left[\gamma^\mu F_1(q^2) + i\,\frac{\sigma^{\mu\nu}q_\nu}{2m_\mu}F_2(q^2)\right]u(p)\,,$$

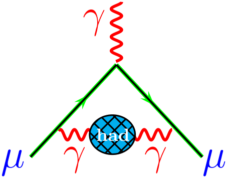
where $F_2(0) = a_\mu$ and the black box represents all orders in perturabation theory.

QED part

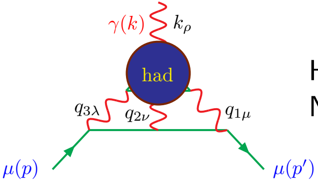


Ex: All QED diagrams of two-loop order (NNLO) in perturbation theory. Diagrams 1-7 represent the universal second order contribution to a_μ, diagram 8 yields the "light", diagram 9 the "heavy" mass dependent corrections.

QCD part

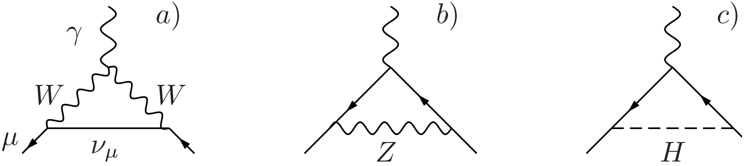


Hadronic vacuum polarization at LO, NLO, NNLO, ... (HVP LO and HO)



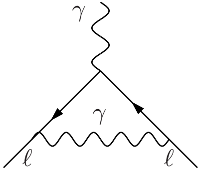
Hadronic light-by-light at LO, NLO, NNLO, ... (HLbL)

Weak part



Ex: All first order (NLO) in perturbation theory contain a) the W boson, b) the Z boson and c) the Higgs (H) boson.

QED 1-Loop

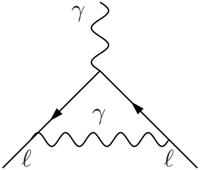


The full vertex Γ^{μ} can be writen as

$$i\Gamma^{\nu} = ie\left(\gamma^{\nu} + \gamma^{\nu}\delta Z_1 + \Lambda^{\nu}_{\text{loop}}\right) \, ,$$

$$\begin{aligned} ie\mu^{\epsilon/2}\Lambda^{\nu}_{\text{loop}}\left(p', p\right) = \mu^{3\epsilon/2} \int \frac{d^d k}{(2\pi)^d} \frac{-ig_{\alpha\beta}}{k^2 + i\varepsilon} \left(ie\,\gamma^{\alpha}\right) \\ \times \frac{i(\not{p}' + \not{k} + m_{\mu})}{(p' + k)^2 - m_{\mu}^2 + i\varepsilon} \left(ie\,\gamma^{\nu}\right) \\ \times \frac{i(\not{p} + \not{k} + m_{\mu})}{(p + k)^2 - m_{\mu}^2 + i\varepsilon} \left(ie\,\gamma^{\beta}\right) \end{aligned}$$

QED 1-Loop



The full vertex Γ^μ can be writen as

$$i\Gamma^\nu = ie\left(\gamma^\nu + \gamma^\nu \delta Z_1 + \Lambda^\nu_{\text{loop}}\right) \ ,$$

$$a_\mu = \frac{m_\mu^2 e^2}{4\pi^2}\int_0^1 dx_1 \int_{x_1}^1 dy \, \frac{y\,(1-y)}{y^2 m_\mu^2} = \frac{e^2}{8\pi^2}$$

$$a_\mu = \frac{\alpha}{2\pi}$$

This is the famous result of Schwinger from 1948 for a massless boson ($m_\gamma \ll m_\mu$).

Theory vs Experiment

Comparison between theory and experimental value of $a_\mu \times 10^{-10}$

Contribution	Value	Error
QED incl. 4-loops + 5-loops	11 658 471.886	0.003
Hadronic LO vacuum polarization	689.46	3.25
Hadronic light–by–light	10.34	2.88
Hadronic HO vacuum polarization	-8.70	0.06
Weak to 2-loops	15.36	0.11
Theory	11 659 178.3	4.3
Experiment	11 659 209.1	6.3
Theory - Experiment	-30.6	7.6

Deviation of $\sim 4\sigma$

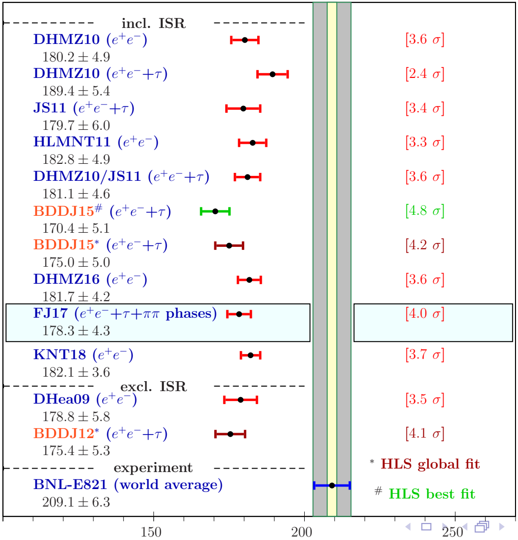
Theory vs Experiment

Comparison between theory and experimental value of $a_\mu \times 10^{-10}$

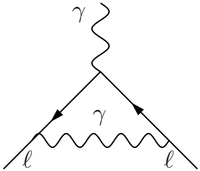
Contribution	Value	Error
QED incl. 4-loops + 5-loops	11 658 471.886	0.003
Hadronic LO vacuum polarization	689.46	3.25
Hadronic light–by–light	10.34	2.88
Hadronic HO vacuum polarization	-8.70	0.06
Weak to 2-loops	15.36	0.11
Theory	11 659 178.3	4.3
Experiment	11 659 209.1	6.3
Theory - Experiment	-30.6	7.6

Deviation of $\sim 4\sigma \longrightarrow$ **New physics?**

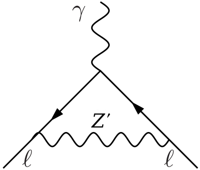
Theory vs Experiment



New Physics at 1-Loop

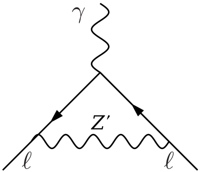


$$\mu^{3\epsilon/2}\int \frac{d^dk}{(2\pi)^d}\frac{-ig_{\alpha\beta}}{k^2-0+i\varepsilon}(-iQ_\mu e\,\gamma^\alpha)\times\frac{i(\not p'+\not k+m_\mu)}{(p'+k)^2-m_\mu^2+i\varepsilon}(-iQ_\mu e\,\gamma^\nu)\times\frac{i(\not p+\not k+m_\mu)}{(p+k)^2-m_\mu^2+i\varepsilon}(-iQ_\mu e\,\gamma^\beta)$$



$$\mu^{3\epsilon/2}\int \frac{d^dk}{(2\pi)^d}\frac{-ig_{\alpha\beta}}{k^2-M_{Z'}^2+i\varepsilon}(-ix_\mu g\gamma^\alpha)\times\frac{i(\not p'+\not k+m_\mu)}{(p'+k)^2-m_\mu^2+i\varepsilon}(-iQ_\mu e\,\gamma^\nu)\times\frac{i(\not p+\not k+m_\mu)}{(p+k)^2-m_\mu^2+i\varepsilon}(-ix_\mu g\gamma^\beta)$$

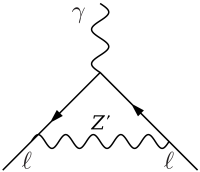
New Physics at 1-Loop



$$a_\mu^{\text{NP}} = \frac{m_\mu^2\, g^2 x_\mu^2}{4\pi^2} \int_0^1 dx_1 \int_{x_1}^1 dy\, \frac{y\,(1-y)}{y^2 m_\mu^2 + (1-y)\, M_{Z'}^2}$$

$$\text{Heavy Boson} \longrightarrow M_{Z'} \gg m_\mu \longrightarrow a_\mu^{\text{NP}} = \frac{1}{12\pi^2} \frac{m_\mu^2}{M_{Z'}^2} g^2 x_\mu^2$$

New Physics at 1-Loop



$$a_\mu^{\text{NP}} = \frac{m_\mu^2\, g^2 x_\mu^2}{4\pi^2} \int_0^1 dx_1 \int_{x_1}^1 dy\, \frac{y\,(1-y)}{y^2 m_\mu^2 + (1-y)\, M_{Z'}^2}$$

$$\text{Heavy Boson} \longrightarrow M_{Z'} \gg m_\mu \longrightarrow a_\mu^{\text{NP}} = \frac{1}{12\pi^2} \frac{m_\mu^2}{M_{Z'}^2} g^2 x_\mu^2$$

- **A new (Z-like type) chargeless vector boson with $\mathbf{M_{Z'}/g \sim 175}$ GeV solve the muon g-2!**

Recall Neutrino Results

Diagonal Dirac-neutrino mass matrix $\mathbf{m}_D$:

Symmetry generator X	$\lvert x_s \rvert$	$\mathbf{M}_R$	$\mathbf{m}_\nu$
$\mathbf{B + L_e - L_\mu - 3L_\tau}$	**2**	$\mathbf{D}_2$	$\mathbf{A}_1$
$B + 3L_e - L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{23} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_e - 6L_\tau$	3		
$B + 9L_e - 3L_\mu - 9L_\tau$	6		
$\mathbf{B + L_e - 3L_\mu - L_\tau}$	**2**	$\mathbf{D}_1$	$\mathbf{A}_2$
$B + 3L_e - 5L_\mu - L_\tau$	2	$(\mathbf{M}_R)_{11} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{23} = 0$	
$B + 3L_e - 6L_\mu$	3		
$B + 9L_e - 9L_\mu - 3L_\tau$	6		
$\mathbf{B - L_e + L_\mu - 3L_\tau}$	**2**	$\mathbf{B}_4$	$\mathbf{B}_3$
$B - L_e + 3L_\mu - 5L_\tau$	2	$(\mathbf{M}_R)_{13} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B + 3L_\mu - 6L_\tau$	3		
$B - 3L_e + 9L_\mu - 9L_\tau$	6		
$\mathbf{B - L_e - 3L_\mu + L_\tau}$	**2**	$\mathbf{B}_3$	$\mathbf{B}_4$
$B - L_e - 5L_\mu + 3L_\tau$	2	$(\mathbf{M}_R)_{12} = (\mathbf{M}_R)_{22} = (\mathbf{M}_R)_{33} = 0$	
$B - 6L_\mu + 3L_\tau$	3		
$B - 3L_e - 9L_\mu + 9L_\tau$	6		

New Physics "Toy Model" Example

Minimal Dark Boson:

New gauge symmetry $U(1)_X$ with 3 ν_R that explains neutrino data, where $\mathbf{X = B + L_e - L_\mu - 3L_\tau}$. Muon charge is $x_\mu = -1$. Then the new (Z') gauge boson with mass given by the VEV of the (S) scalar, $M_{Z'} \sim g\langle S \rangle \lesssim 200$ GeV can:

- Explain the muon g-2 (a_μ)
- Be a viable dark matter candidate!

New Physics "Toy Model" Example

Minimal Dark Boson:

New gauge symmetry $U(1)_X$ with 3 ν_R that explains neutrino data, where $\mathbf{X} = \mathbf{B} + \mathbf{L_e} - \mathbf{L}_\mu - \mathbf{3L}_\tau$. Muon charge is $x_\mu = -1$. Then the new (Z') gauge boson with mass given by the VEV of the (S) scalar, $M_{Z'} \sim g\langle S \rangle \lesssim 200$ GeV can:

- Explain the muon g-2 (a_μ)
- Be a viable dark matter candidate!

Do not work.

Sketch of New Physics "Toy Models"

What (may) work:

- **Dark Scalar**

Same model as before but with two singlet scalars (S), one of them serves as the DM candidate while the other acquires VEV to "give" masses. Besides explaining the neutrino data it also explain DM phenomenology and the muon g-2.

- **Dark Photon**

Same model as before but with much smaller gauge boson mass ($M_{Z'} \lesssim 1$ GeV) and coupling ($g \lesssim 10^{-4}$).

- **More complex models**

Sterile neutrinos with Inverse Seesaw, Dark axion, Extended models with VL fermion as DM candidate...

Good Mood Conclusions

- I shouldn't bother you with all that! (I should)
- To experimentalists: Stop giving us data! Or then, give us much more data!
- More serious: Seesaw mechanism combined with neutrino CP-phase violation can explain BAU. So these models can explain several unresolved problems in physics: The origin of matter, the muon g-2 anomaly, dark matter and even neutrino data!

Inverse Seesaw

When the scalar fields acquire v.e.v. we obtain the mass terms

$$\mathbf{m}_u\, \overline{u_L} u_R + \mathbf{m}_d\, \overline{d_L} d_R + \mathbf{m}_\ell\, \overline{e_L} e_R + \frac{1}{2}\mathcal{M}\, \Psi^T C\, \Psi + \text{H.c.},$$

where $\Psi = (\nu_L^c, \nu_R, \nu_S)^T$ and

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_{DS} \\ m_D^\intercal & M_R & M_{RS} \\ m_{DS}^\intercal & M_{RS}^\intercal & M_S \end{pmatrix}.$$

In the limit where $M_S \ll m_D \ll M_{RS}$, the inverse seesaw formula for the effective neutrino mass matrix is given by

$$m_\nu = -m_D\, \left(M_{RS}\, M_S^{-1} M_{RS}^\intercal\right)^{-1} m_D^\intercal\,,$$

after the matrix is diagonalized.

Solutions

With two scalar singlets there are cases (indicated with *) that are compatible with $M_R = 0$.

N_R	N_S	a	Glashow(A_1-C)	viable (#0< 3)
2	2	0	no	no
2	2	1	no	no
3	2	0	no	yes*
3	2	1	no	yes*
2	3	0	no	no
2	3	1	no	no
3	3	0	yes	yes*
3	3	1	yes*	yes*

Now we need to study/look to Z' and S_i phenomenology, identify the symmetries that lead to viable patterns and check their compatibility with data through MINUIT minimization.

Concluding Remarks

- We randomize the elements of $\mathbf{m}_\nu$ for several textures with equal entries, then diagonalise through $\mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu^*$ and check whether the observables reconstructed from $\mathbf{U}_\nu$ are compatible with data at $1\sigma/3\sigma$ CL.
- Many compatible classes of textures with increased predictability were found. Indeed, three textures with only three physical parameters tightly constrain the lightest neutrino mass, $m_{\beta\beta}$ and the complex phases (δ, α_{21} and α_{31}).
- An interesting possibility is to see if such predictive textures could be generated in different Seesaw scenarios or from a symmetry principle.
- Further studies and data are needed to discriminate, among the viable neutrino mass matrix textures, the patterns that more appropriately describe the observations.

General Strategy

Use the MINUIT package to minimize the function

$$\chi^2(x) = \sum_i \frac{(\mathcal{P}_i(x) - \overline{\mathcal{O}}_i)^2}{\sigma_i^2}.$$

The $\mathbf{m}_\nu$ matrix elements are denoted by x, $\mathcal{P}_i(x)$ are the predictions of the *Ansätze* (each texture in each class) for the observables $\mathcal{O}_i$, $\overline{\mathcal{O}}_i$ are the best-fit values of $\mathcal{O}_i$, and σ_i are their corresponding $1\sigma/3\sigma$ errors.

A given pattern is only considered compatible at $1\sigma/3\sigma$ level if each observable (Δm_{ij}^2, θ_{ij}, and δ) fits in its $1\sigma/3\sigma$ interval.

Main Goal

In the flavour basis, we look for other neutrino textures which have the same predictability or which are even more predictive (contain less than five physical parameters, $n \leqslant 5$) and can accommodate the neutrino oscillation data.

$$n = 5 \longrightarrow \mathrm{T}^{6}_{12} : \begin{pmatrix} a & a & * \\ a & * & * \\ * & * & 0 \end{pmatrix};$$

$$n = 4 \longrightarrow \mathrm{T}^{2}_{34,56} : \begin{pmatrix} * & 0 & a \\ 0 & a & b \\ a & b & b \end{pmatrix}, \; n = 3 \longrightarrow \mathrm{T}^{2,3}_{145} : \begin{pmatrix} a & 0 & 0 \\ 0 & a & a \\ 0 & a & * \end{pmatrix}$$

Textures with Three Parameters

$$\text{NO} \longrightarrow \; \text{T}^{2}_{136,45} : \begin{pmatrix} a & 0 & a \\ 0 & b & b \\ a & b & a \end{pmatrix};$$

$$\text{IO} \longrightarrow \; \text{T}^{4}_{123,56} : \begin{pmatrix} a & a & a \\ a & 0 & b \\ a & b & b \end{pmatrix}, \; \text{T}^{6}_{123,45} : \begin{pmatrix} a & a & a \\ a & b & b \\ a & b & 0 \end{pmatrix}.$$

Textures with Three Parameters

Neutrino observables are quite constrained in the three cases, some can even be approximately predicted:

Spectrum	Texture	δ/π	m [meV]	$m_{\beta\beta}$ [meV]	α_{21}/π	α_{31}/π
NO	$T^2_{136,45}$	0.38 – 0.45	11 – 13	9.9 – 11	≈ 1.5	0.66 – 0.75
IO	$T^4_{123,56}$	0.26 – 0.38	9.7 – 11	27 – 30	1.3 – 1.4	0.39 – 0.56
	$T^6_{123,45}$	0.62 – 0.74	9.7 – 10	27 – 30	0.63 – 0.72	1.4 – 1.6

Since the complex matrix elements are free, there is an internal symmetry $\delta \to -\delta$, $\alpha_{21} \to -\alpha_{21}$ and $\alpha_{31} \to -\alpha_{31}$. This symmetry and the ones between $T^4_{123,56}$ and $T^6_{123,45}$ explain the very similar predictions.

Model and Motivation

Abelian extension of the SM based on an extra $U(1)_X$ gauge symmetry, with $X \equiv a\,B - \sum_\alpha b_\alpha L_\alpha$ being an arbitrary linear combination of the baryon number B and the individual lepton numbers L_α.

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$$

Now in our inverse seesaw framework the extra matter content is: singlet right-handed neutrinos ν_R, sterile neutrinos ν_S and complex singlet scalars S_i.

Masses

The Lagrangian terms that generate the SM fermion masses and are compatible with inverse seesaw for Majorana neutrinos are given by

$$
\begin{aligned}
&\mathbf{Y}_u\, \overline{q_L} u_R \widetilde{H} + \mathbf{Y}_d\, \overline{q_L} d_R H + \mathbf{Y}_e\, \overline{\ell_L} e_R H \\
&+ \mathbf{Y}_R\, \overline{\ell_L} \nu_R \widetilde{H} + \mathbf{Y}_S\, \overline{\ell_L} \nu_S \widetilde{H} \\
&+ \frac{1}{2}\mathbf{m}_R\, \nu_R^T C \nu_R + \nu_R^T C \nu_R \sum_i (\mathbf{Y}_{1i}\, S_i + \mathbf{Y}_{2i}\, S_i^*) \\
&+ \frac{1}{2}\mathbf{m}_{RS}\, \nu_R^T C \nu_S + \nu_R^T C \nu_S \sum_i (\mathbf{Y}_{3i}\, S_i + \mathbf{Y}_{4i}\, S_i^*) \\
&+ \frac{1}{2}\mathbf{m}_S\, \nu_S^T C \nu_S + \nu_S^T C \nu_S \sum_i (\mathbf{Y}_{5i}\, S_i + \mathbf{Y}_{6i}\, S_i^*)
\end{aligned}
$$

$+$H.c.,

Inverse Seesaw

When the scalar fields acquire v.e.v. we obtain the mass terms

$$\mathbf{m}_u \, \overline{u_L} u_R + \mathbf{m}_d \, \overline{d_L} d_R + \mathbf{m}_\ell \, \overline{e_L} e_R + \frac{1}{2} \mathcal{M} \, \Psi^T C \, \Psi + \text{H.c.},$$

where $\Psi = (\nu_L^c, \nu_R, \nu_S)^T$ and

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & m_{DS} \\ m_D^\intercal & M_R & M_{RS} \\ m_{DS}^\intercal & M_{RS}^\intercal & M_S \end{pmatrix} .$$

In the limit where $M_S \ll m_D \ll M_{RS}$, the inverse seesaw formula for the effective neutrino mass matrix is given by

$$m_\nu = -m_D \, \left(M_{RS} \, M_S^{-1} M_{RS}^\intercal \right)^{-1} m_D^\intercal \, ,$$

after the matrix is diagonalized.

Solutions

With two scalar singlets there are cases (indicated with *) that are compatible with $M_R = 0$.

N_R	N_S	a	Glashow(A_1-C)	viable (#0< 3)
2	2	0	no	no
2	2	1	no	no
3	2	0	no	yes*
3	2	1	no	yes*
2	3	0	no	no
2	3	1	no	no
3	3	0	yes	yes*
3	3	1	yes*	yes*

Now we need to study/look to Z' and S_i phenomenology, identify the symmetries that lead to viable patterns and check their compatibility with data through MINUIT minimization.

Concluding Remarks

- We randomize the elements of $\mathbf{m}_\nu$ for several textures with equal entries, then diagonalise through $\mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu^*$ and check whether the observables reconstructed from $\mathbf{U}_\nu$ are compatible with data at $1\sigma/3\sigma$ CL.
- Many compatible classes of textures with increased predictability were found. Indeed, three textures with only three physical parameters tightly constrain the lightest neutrino mass, $m_{\beta\beta}$ and the complex phases (δ, α_{21} and α_{31}).
- An interesting possibility is to see if such predictive textures could be generated in different Seesaw scenarios or from a symmetry principle.
- Further studies and data are needed to discriminate, among the viable neutrino mass matrix textures, the patterns that more appropriately describe the observations.