White dwarfs in an ungravity-inspired model

Hodjat Mariji

in collaboration with

Orfeu Bertolami

Centro de Física do Porto, Departamento de Física e Astronomia,
Faculdade de Ciências da Universidade do Porto

Phys. Rev. D ***93***,*104046 (2016)*

- ❑ **Ungravity (UG) Model**
- ❑ **UG Equilibrium Equations**
- ❑ **WD’s Equilibrium Equations**
- ❑ **Results & Discussion**

❑ Ungravity (UG) Model

UG arises from the coupling between spin-2 "*unparticle*" to the stress tensor!

$$T^{\mu\nu} \longrightarrow T^{\mu\nu} + T^{\mu\nu}_{\mathcal{U}} \quad , \quad T^{\mu\nu}_{\mathcal{U}} \sim \sqrt{|g|}\, T^{\alpha\beta} \mathcal{O}^{\mathcal{U}}_{\alpha\beta}\, g_{\mu\nu}$$

[H. Goldberg and P. Nath, PRL **100**, 031803 (2008)]

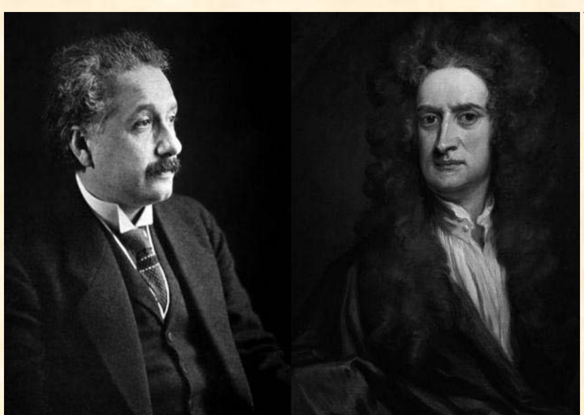
Unparticle?

In order to compensate the *lack of scale invariance* at the low energy sector of the Standard Model, **Howard Georgi** presented an appealing idea by introducing a type of Stuff, dubbed "**Unparticle**".

Howard Georgi:

[H. Georgi, PRL **98**, 221601 (2007); PLB **650**, 275 (2007)]

" I found a scheme in which this may be possible by keeping the unparticle world and the world of Standard Model particles separate from one another except at very high energies."



Universal Gravitation Law $F_N = G_N \dfrac{MM'}{r^2}$

General Relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4} T_{\mu\nu}$$

need to be modified for **Unparticle Physics**

Ungravitational potential

Lowest order corrections ($g^{\mu\nu} \rightarrow \eta^{\mu\nu}$) of ungravity alters the classical laws of gravitation

Newtonian ungravitational potential

$$\phi_*(r) = -\frac{G_* M}{r}\left[1 + \left(\frac{R_*}{r}\right)^{\alpha-1}\right]$$

$$G_* = \frac{G}{1 + \left(\frac{R_*}{R_0}\right)^{\alpha-1}}, \quad G_* \simeq G/2$$

$$\alpha = 2d_u - 1$$

[O. Bertolami, *et. al*, PRD **80**, 022001 (2009)]

Ungravitational Constant

Characteristic Length Scale of UG

Scaling Dimension of UG operator

❑ WD's Equilibrium Equations

The most general hydrostatic equilibrium equation (TOV)

$$4\pi r^2 dP(r) = -\frac{GM(r)dM(r)}{r^2}\left[1 + \frac{P(r)}{\rho(r)c^2}\right]\left[1 + \frac{4\pi r^3 P(r)}{M(r)c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}$$

$$dM(r) = 4\pi\rho(r)r^2 dr$$

For a WD ⟹ $\frac{dP(r)}{dr} = -\frac{GM(r)\rho(r)}{r^2}$ ⟹ $\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP(r)}{dr}\right) = -4\pi G\rho(r)$

NHE equation

o EoS for WD

$T_{int} \sim 10^7\ K \Rightarrow$ WDs are too cold to ignite nuclear reactions

Bounded nucleons contribute to all the WD energy density ($\mu_e n_e m_H c^2$, $\mu_e = A/Z$)

Pressure of degenerate electrons (rather than temperature) supports a WD against gravitational collapse!

• *Polytropic gas*

$$P = K\rho^{(n+1)/n}$$

n: polytropic index

$$\begin{cases} \rho = \rho_c \theta^n, \\ r = \beta_p \xi, \\ \beta_p = \left[\frac{(n+1)K}{4\pi G}\rho_c^{(1-n)/n}\right]^{1/2} \end{cases}$$

NHE equation $\Longrightarrow$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\theta^n$$

(Lane-Emden (**LE**))

$$\begin{cases} \theta(\xi = 0) = 1 \\ \theta'(\xi = 0) = 0 \end{cases}$$

$$M(\xi_{10}) = 4\pi\rho_c\beta_p^3\left(-\xi^2\frac{d\theta}{d\xi}\right)\Big|_{\xi=\xi_{10}}$$

$$R = \beta_p \xi_{10}$$

ξ_{10} : *first zero of the LE equation solution*

• *Degenerate gas*

$$E_{F,e} = \sqrt{(p_{F,e}c)^2 + E_{0,e}^2} \gg E_{0,e} \Rightarrow T_{int} \ll T_{F,e}$$

→ WDs satisfy degeneracy condition

Fermi-Dirac Distribution Function

$$F(\mathcal{E}) = \frac{1}{\exp[(\mathcal{E} - \mathcal{E}_F)/kT] + 1} \Rightarrow F(\mathcal{E}) \to \Theta(\mathcal{E} - \mathcal{E}_F)$$

$$P = \frac{1}{3\pi^2\hbar^3}\int_0^{p_F} \frac{p^2}{\sqrt{m^2+\frac{p^2}{c^2}}} p^2 dp = Af(x)$$

$$A \simeq 6.002 \times 10^{22} erg/cm^3$$

$$f(x) = x(2x^2 - 3)(x^2 + 1)^{1/2} + 3sinh^{-1}(x)$$

$$\rho = Bx^3, \quad B \simeq 9.74 \times 10^5 \mu_e \ g/cm^3$$

$$x = p_F/m_e c$$

• *Degenerate gas*

NHE equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dX}{dr}\right) = -\frac{\pi G B^2}{2A}x^3 \qquad X = \sqrt{x^2+1}$$

$$X = X_c\Phi,$$

$$X_c = (x_c^2+1)^{1/2},$$

$$\xi = r/\beta_d$$

$$\beta_d = \sqrt{\frac{2A}{\pi G B^2 X_c^2}}$$

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right) = -(\Phi^2 - X_c^2)^{\frac{3}{2}}$$

(**LE** equation)

$$X(\xi_{10}) = 1, \quad \Phi(\xi = 0) = 1, \quad \Phi'(\xi = 0) = 0$$

$$M(\xi_{10}) = 4\pi B X_c^3 \beta_d^3 \left(-\xi^2\frac{d\Phi}{d\xi}\right)\Big|_{\xi_{10}} \qquad R = \beta_d \xi_{10}$$

❑ UG Equilibrium Equations

$$\phi_*(r) = -\frac{G_* M}{r}\left[1 + \left(\frac{R_*}{r}\right)^{\alpha-1}\right]$$

$$\vec{F}_* + \vec{F}_P\Big|_{H.E.} = 0$$

$$\frac{dP(r)}{dr} = -\frac{G_* M(r)\rho(r)}{r^2}\left[1 + \left(\frac{R_*}{r}\right)^{\alpha-1}\right]$$

$$\frac{1}{r^2}\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP(r)}{dr}\right) = -4\pi G_* \rho(r)\left[1 + \alpha\left(\frac{R_*}{r}\right)^{\alpha-1}\right] + \frac{G_* M(r)}{R_*^3}\left[\alpha(\alpha-1)\left(\frac{R_*}{r}\right)^{\alpha+2}\right]$$

UG-LE Equation for WD

polytropic gas model

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) = -\frac{G_*}{G}\left\{\left[1 + \alpha\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\right]\theta^n + \left[\alpha(\alpha-1)\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\left(\frac{1}{\xi}\frac{d\theta}{d\xi}\right)\right]\right\}$$

degenerate gas model

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\Phi}{d\xi}\right) = -\frac{G_*}{G}\left\{\left[1 + \alpha\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\right](\Phi^2 - X_c^2)^{\frac{3}{2}} + \left[\alpha(\alpha-1)\left(\frac{\xi_*}{\xi}\right)^{\alpha-1}\left(\frac{1}{\xi}\frac{d\Phi}{d\xi}\right)\right]\right\}$$

❑ Results & Discussion

We select that solution of the UG-LE equations (for both approaches) for which the calculated M, R, and L remain within the observational range

Sirius B (SIB) and e-Reticulum (HDB)

WD	$(M_0 \pm \triangle M_0)/M_S$	$(R_0 \pm \triangle R_0)/R_S$	$T_{eff} \pm \triangle T_{eff}(K)$	$(L_0 \pm \triangle L_0)/L_S$
SIB	1.02±0.02	0.0081±0.0002	25193±37	0.0237±0.0013
HDB	0.616±0.022	0.0129±0.0003	15310±350	0.0082±0.0011

[SIB] M. A. Barstow, *et. al*, Mon. Not. R. Astron. Soc. **362**, 1134 (2005)]

[HDB] J. Farihi, *et. al*, Mon. Not. R. Astron. Soc. **417**, 1735 (2011)

$$L = 4\pi R^2 \sigma T_{eff}^4$$

First: the best solutions of the usual LE equations

Inputs for SIB (HDB):

$$\rho_c = 3.20 \times 10^7 (3.22 \times 10^6) g/cm^3$$

$$n = 2.03(1.73)$$

Model	WD	M_{10}/M_S	R_{10}/R_S	L_{10}/L_S
Degenerate	SIB	1.0988	0.0080	0.0231
	HDB	0.6012	0.0127	0.0079
Polytropic	SIB	1.0201	0.0081	0.0237
	HDB	0.6162	0.0129	0.0082

For the UG-LE equations, by keeping the same inputs:

For each α we have we have a range for R_* ($[R_*^-, R_*^+]$) corresponding to $M \pm \Delta M$.

A few constraints on the UG-LE solutions:

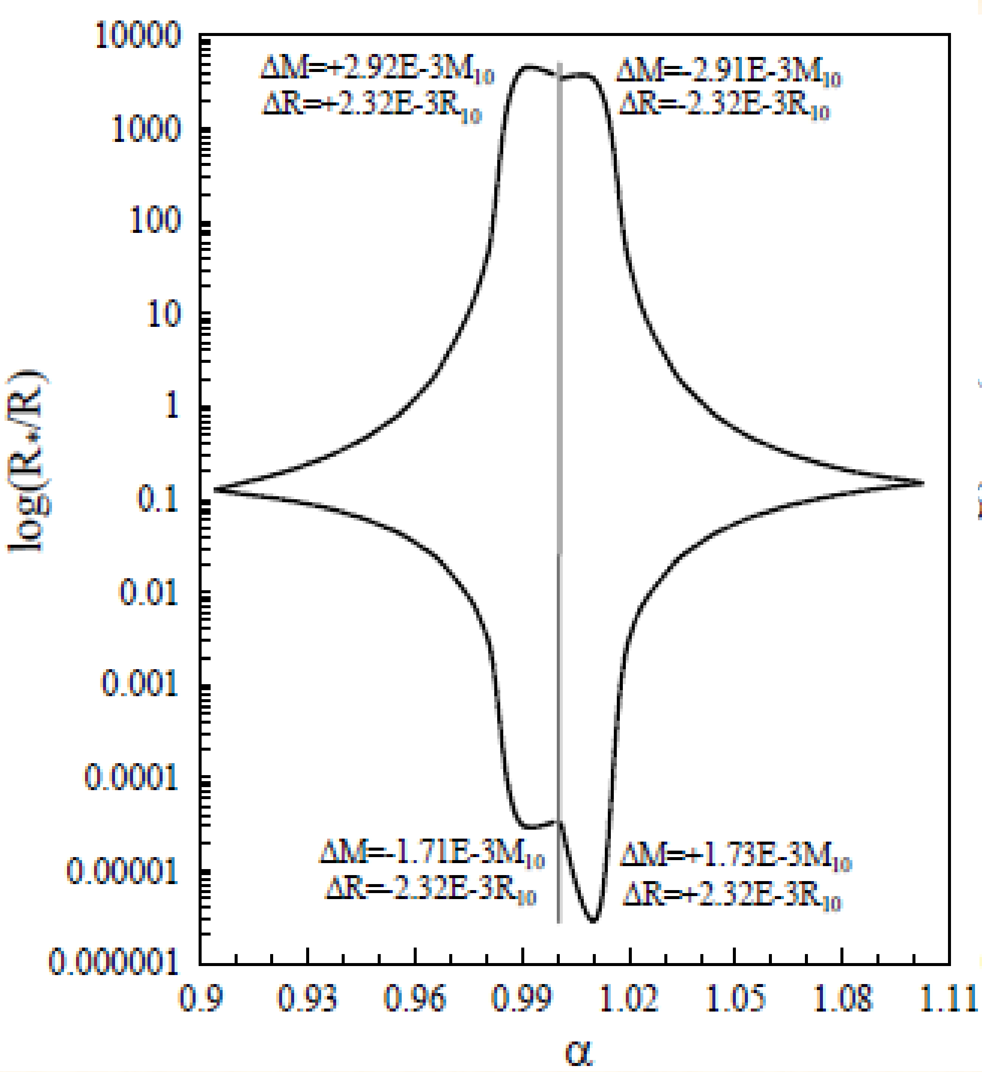
So, for each α, those values of R_* are acceptable which stay within that range.

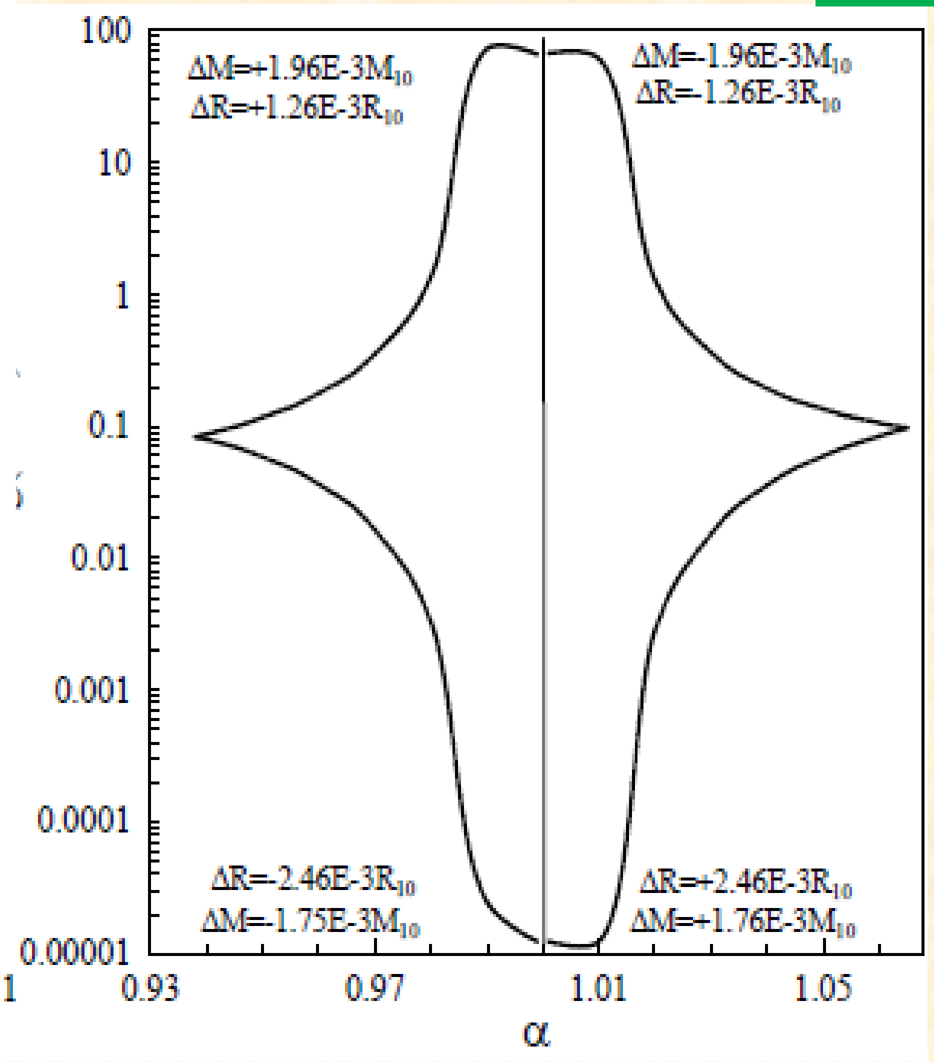
In order to find the allowed region for α and R_*, we compute R_*^+ and R_*^-, the upper and lower bound on R_*, respectively, by setting the upper and lower values of M (calculated at ξ_{10}^*) so that the values of R and L remain within the observational range.

In each portion of the allowed regions we set a fixed value for the uncertainty in M, R, and L:

$$\triangle M = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right)^2 \left(\frac{\eta_{11}'}{\eta_{10}'}\right) - 1\right] M_{10}, \quad \triangle R = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right) - 1\right] R_{10}, \quad \triangle L = \left[\left(\frac{\xi_{10}^*}{\xi_{10}}\right)^2 - 1\right] L_{10},$$

η' indicates $\theta'(\Phi')$



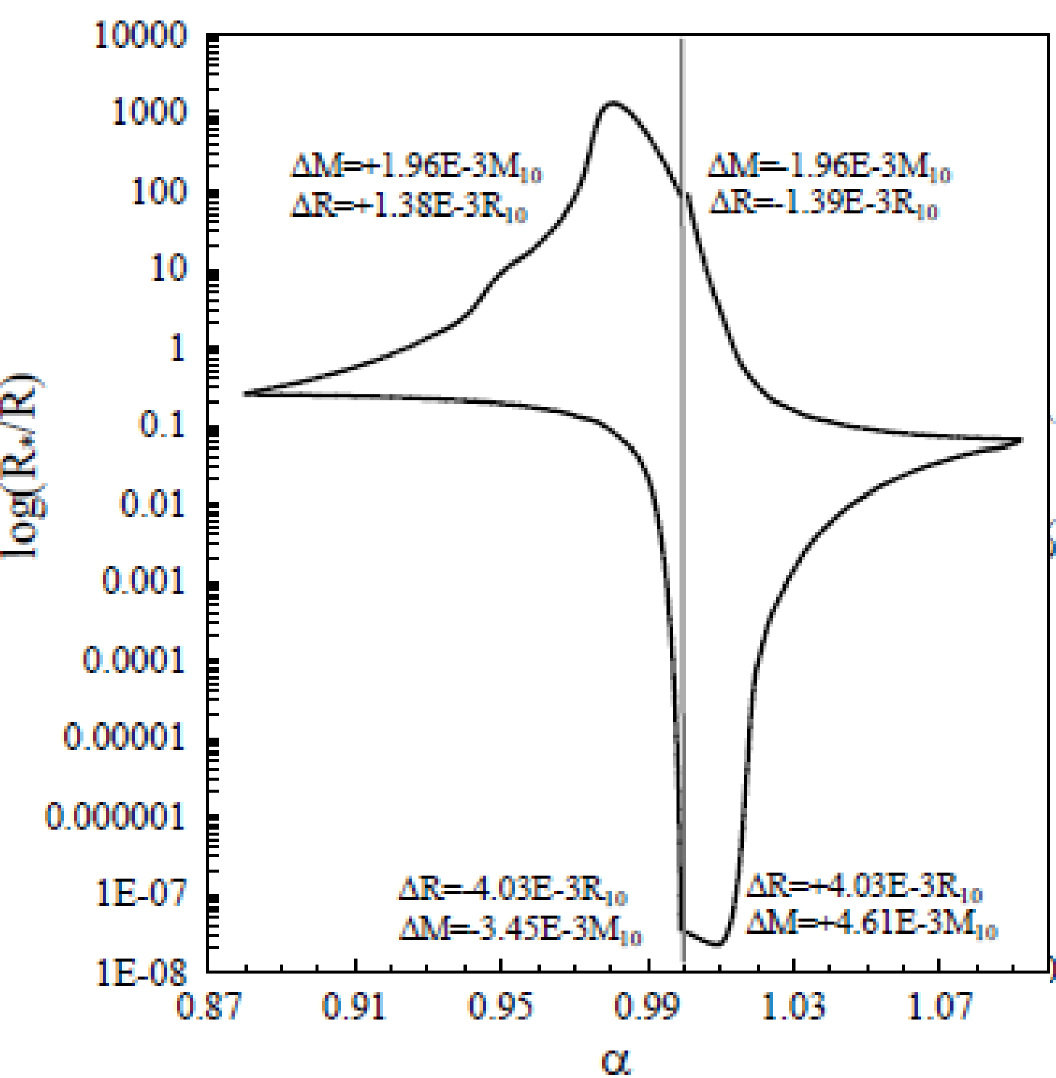


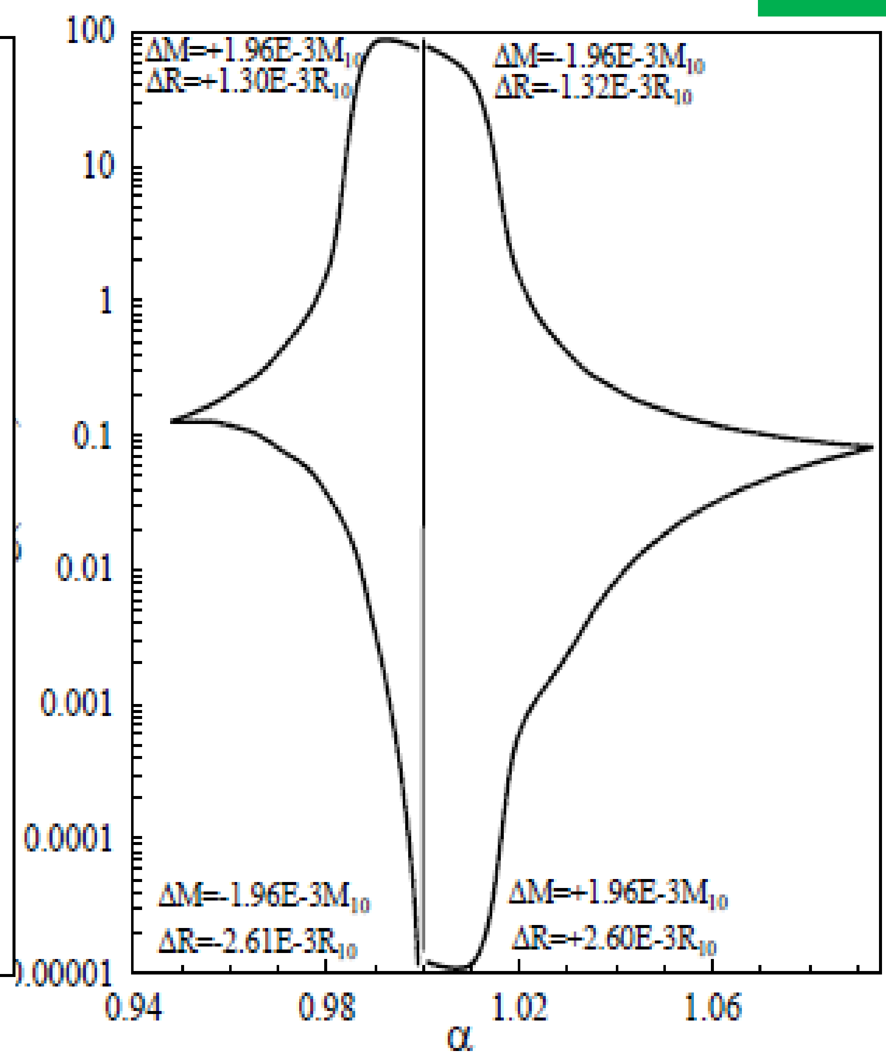
HDB **SIB**

(Polytropic)

The allowed region for the UG parameters

The characteristic length has been normalized by R, the radius of the relevant WD





HDB **SIB**

(Degenerate)

The allowed region for the UG parameters

The characteristic length has been normalized by R, the radius of the relevant WD

Model	WD	α	$R_*(m)$	M/M_S	R/R_S	L/L_S
Degenerate	SIB	0.948	713.707	1.040	0.0079	0.0226
		1.093	460.951	1.000	0.0083	0.0248
	HDB	0.880	2261.582	0.638	0.0126	0.0078
		1.092	581.410	0.594	0.0132	0.0858
Polytropic	SIB	0.942	445.632	1.038	0.0079	0.0226
		1.065	550.077	1.000	0.0083	0.0248
	HDB	0.904	1141.932	0.638	0.0126	0.0078
		1.102	1345.948	0.594	0.0132	0.0858

By increasing the ratio M/R, the allowed region for the UG parameters becomes smaller.

For example, when the ratio M/R increases about 2.5 times, α gets closer to unity by about 4% and R_* gets reduced by 60% based on the limit values of α for the polytropic model.

UG effect on the Chandrasekhar mass limit M_{Ch}

$$\begin{cases} M_{Ch} = 0.721\,(-\xi^2\theta')\,|_{\xi_{10}}\,M_S \\ \xi_{10} = 6.89679 \text{ and } \theta'_{10} = -0.04243. \end{cases} \Rightarrow M_{Ch} = 1.45 M_S$$

But there are WDs with masses greater than M_{Ch} such as WD 1143+321 (***M=1.52 M_S***)

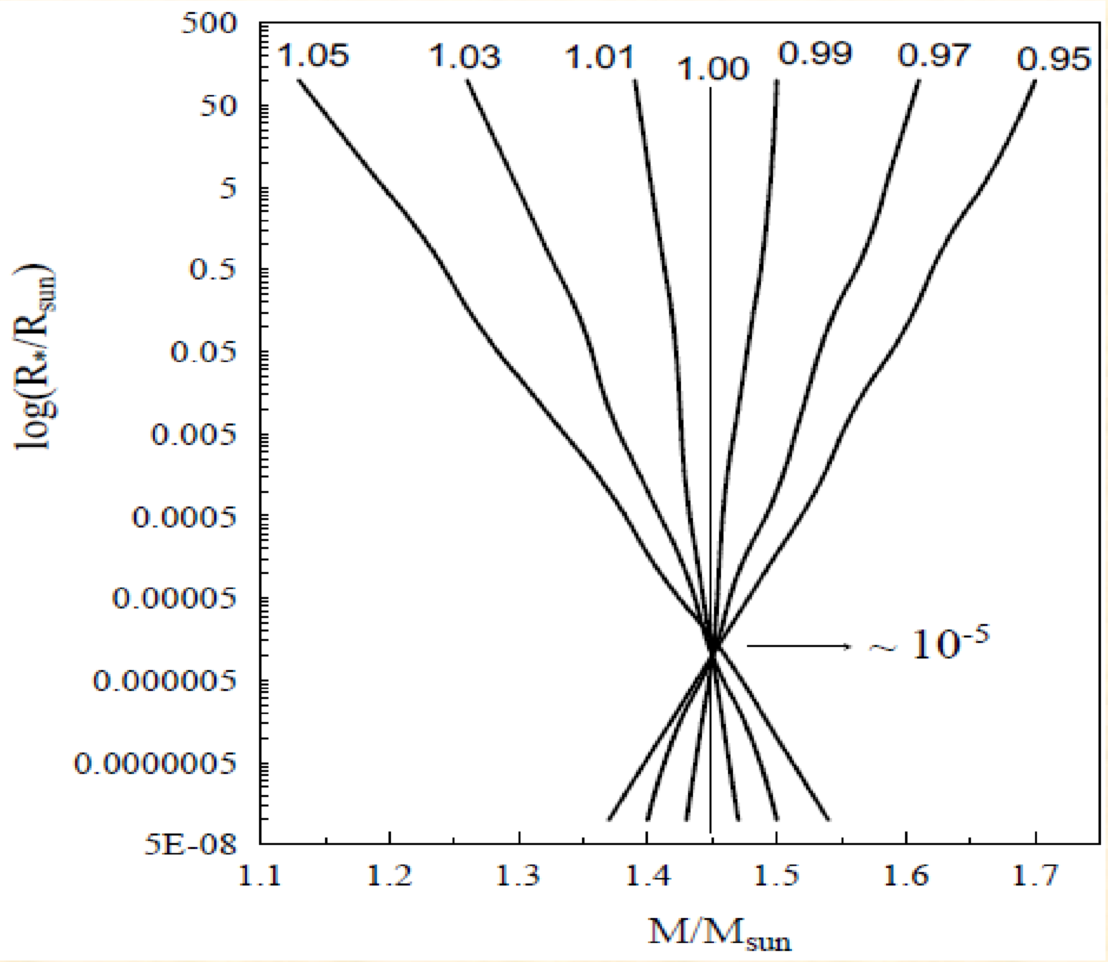
[N. K. Glendenning, *Compact Stars: Nuclear Physics, Particle Physics, and General Relativity* (Springer-Verlag, New York, 2000)]

Don't worry **;D**

The existence of these WDs can be accommodated within the UG model

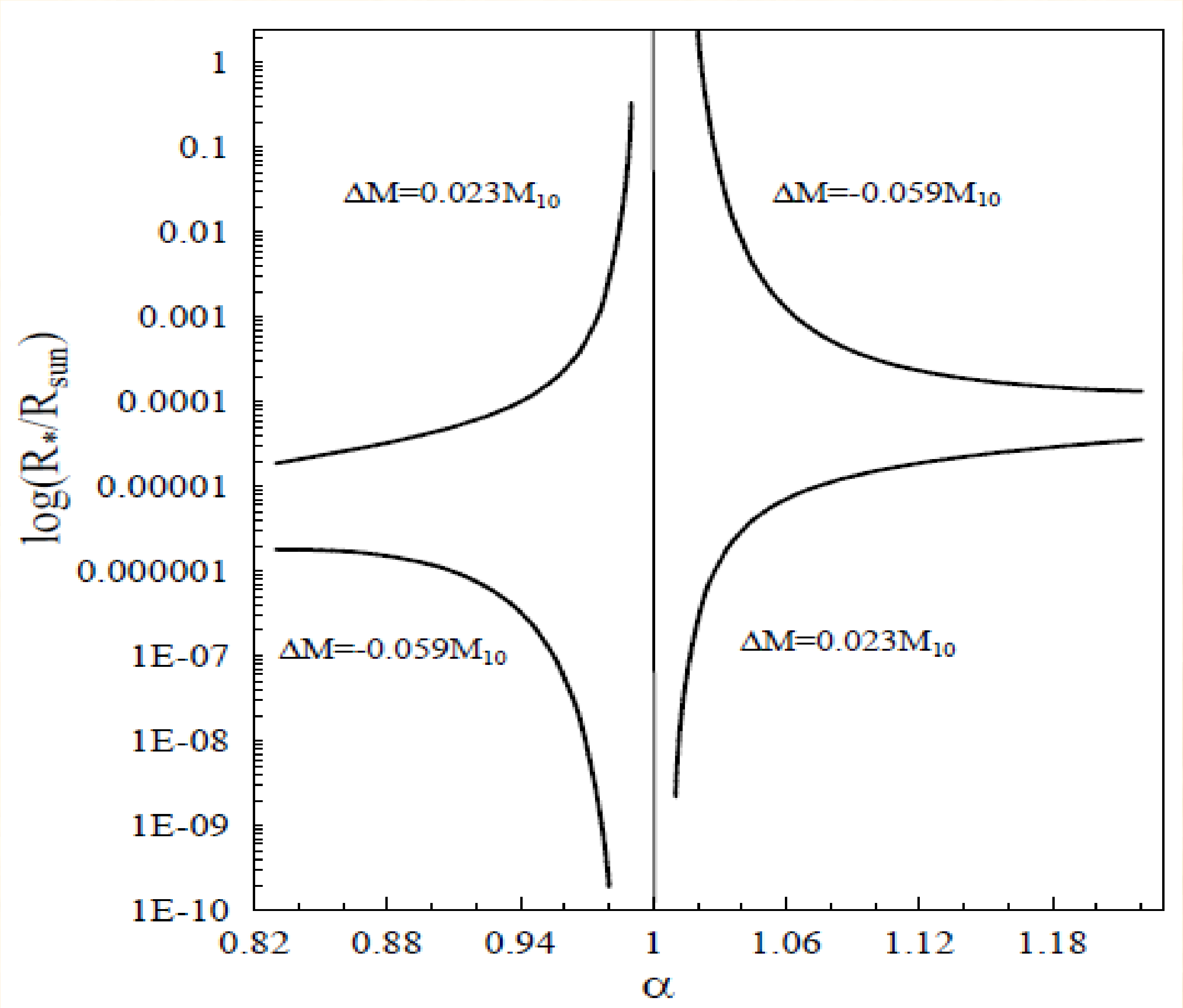
The curves rotate (counter) clockwise around a point (M_{Ch}, $R_*\sim 7\ km$) for $\alpha \to 1^{(-)+}$

So, one can recover M_{Ch} independently of the characteristic length of UG for $R_*\sim 7\ km$!



Ultra-massive WDs (M > 1.1 M_S**)**

EUVE J1746-706. $M = 1.43 M_S$ $\Delta M = 0.06 M_S$ UG polytropic gas model ($n = 3$)

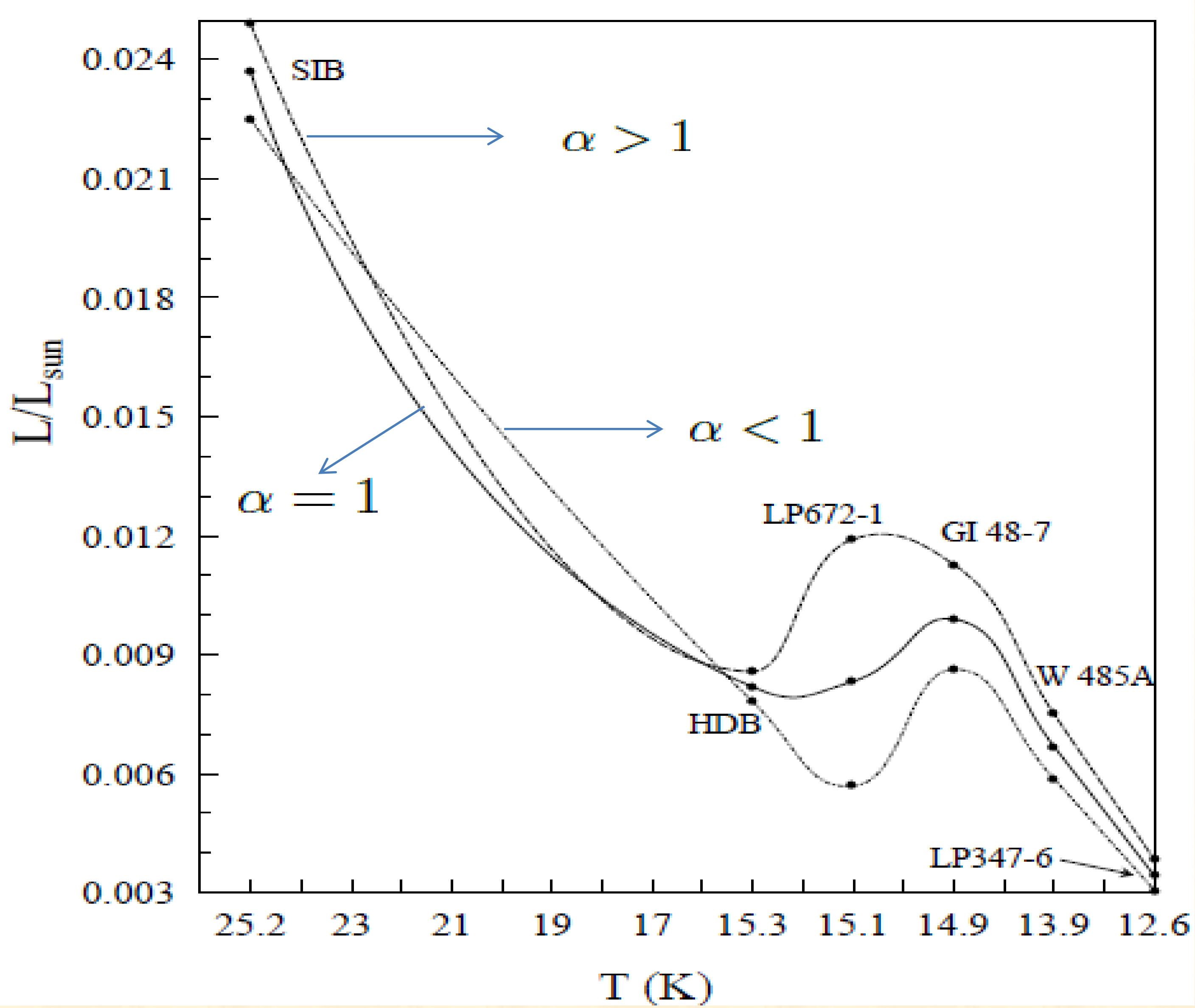


It seems that we should include general relativity corrections in the UG-LE equation

UG effect on the location of WDs in H-R diagram

WD	Alt ID	$M_0 \pm \triangle M$	$R_0 \pm \triangle R$	$T_{eff} \pm \triangle T_{eff}(K)$	α	$R_*(m)$
0642-166	Sirius B	1.02±0.02	0.0081±0.0002	25193±37	0.975	278.5
					1.113	592
0416-594	ε Ret B	0.62±0.022	0.0129±0.0003	15310±350	0.917	1178.8
					1.089	1366.8
1105-048	LP 672-1	0.45±0.094	0.0133±0.0026	15141±88	0.530	548.7
					1.380	347.4
1143+321	G148-7	0.71±0.072	0.0149±0.0010	14938±96	0.768	1124.5
					1.255	1845.2
1327-083	W485	0.53±0.079	0.0141±0.00085	13920±167	0.846	17338
					1.305	2489.3
2341+322	LP 347-6	0.56±0.022	0.0124±0.0007	12300±148	0.790	1039.6
					1.230	1573.6

UG effect on the location of WDs in H-R diagram



Thanks for your attention