Cosmology in generalized hybrid metric-Palatini gravity

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Motivation

Fact: Universe is expanding at an accelerated rate.[1, 2]

Cosmological constant (Λ) vs. Modified gravity[3, 4, 5]

Hybrid metric-Palatini gravity[6, 7]:

1. Passes the solar system observational constraints;
2. Long range scalar field modifies cosmological dynamics;
3. Stable against perturbations.

Next step: **Generalized** hybrid metric-Palatini gravity[8]

$$\mathcal{L}_H = \frac{\sqrt{-g}}{2\kappa^2}\left[R + f\left(\mathcal{R}\right)\right] \quad \rightarrow \quad \mathcal{L}_{GH} = \frac{\sqrt{-g}}{2\kappa^2} f\left(R, \mathcal{R}\right). \tag{1}$$

Action and field equations

Generalized hybrid metric-Palatini gravity:

$$S = \frac{1}{2\kappa^2}\int_\Omega \sqrt{-g} f\left(R, \mathcal{R}\right) d^4x + S_m, \tag{2}$$

$$\mathcal{R} \equiv g_{ab}\mathcal{R}^{ab}, \qquad \mathcal{R}_{ab} = \partial_c \hat{\Gamma}^c_{ab} - \partial_b \hat{\Gamma}^c_{ac} + \hat{\Gamma}^c_{cd}\hat{\Gamma}^d_{ab} - \hat{\Gamma}^c_{ad}\hat{\Gamma}^d_{cb}. \tag{3}$$

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Applying the variational method with respect to $\hat{\Gamma}$

$$\hat{\nabla}_c \left(\sqrt{-g}\frac{\partial f}{\partial \mathcal{R}} g^{ab}\right) = 0. \tag{4}$$

Implies that it exists a new metric $h_{ab} = g_{ab}\partial f/\partial\mathcal{R}$ such that

$$\hat{\Gamma}^a_{bc} = \frac{1}{2} h^{ad}\left(\partial_b h_{dc} + \partial_c h_{bd} - \partial_d h_{bc}\right). \tag{5}$$

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The field equations in this theory are

$$\frac{\partial f}{\partial R}R_{ab}+\frac{\partial f}{\partial\mathcal{R}}\mathcal{R}_{ab}-\frac{1}{2}g_{ab}f\left(R,\mathcal{R}\right)-\left(\nabla_a\nabla_b-g_{ab}\Box\right)\frac{\partial f}{\partial R}=\kappa^2T_{ab}. \tag{6}$$

Scalar-tensor representation

Adding two auxiliary fields α and β to the action

$$S = \frac{1}{2\kappa^2} \int_\Omega \sqrt{-g} \left[f(\alpha, \beta) + \frac{\partial f}{\partial \alpha} (R - \alpha) + \frac{\partial f}{\partial \beta} (\mathcal{R} - \beta) \right] d^4x. \quad (7)$$

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Using $\alpha = R$ and $\beta = \mathcal{R}$ we recover the original action. Defining:

$$\varphi = \frac{\partial f}{\partial \alpha} \qquad \psi = -\frac{\partial f}{\partial \beta} \tag{8}$$

$$S = \frac{1}{2\kappa^2}\int_\Omega \sqrt{-g}\left[\varphi R - \psi \mathcal{R} - V\left(\varphi,\psi\right)\right] d^4x, \tag{9}$$

$$V\left(\varphi,\psi\right) = -f\left(\alpha,\beta\right) + \varphi\alpha - \psi\beta. \tag{10}$$

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The relation between the Ricci tensors is given by

$$\mathcal{R} = R + \frac{3}{\psi^2} \partial^a \psi \partial_a \psi - \frac{3}{\psi} \Box \psi. \tag{11}$$

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The action in the form of a scalar-tensor action becomes

$$S = \frac{1}{2\kappa^2}\int_\Omega \sqrt{-g}\left[\left(\varphi-\psi\right)R - \frac{3}{2\psi}\partial^a\psi\partial_a\psi - V\left(\phi,\psi\right)\right]d^4x. \tag{12}$$

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Applying the variational method, the equations of motion are

$$\begin{aligned}\left(\varphi-\psi\right)G_{ab}=\kappa^2T_{ab}+\frac{3}{2\psi}\partial_a\psi\partial_b\psi+\nabla_a\nabla_b\left(\varphi-\psi\right)-\\-\left(\Box\left(\varphi-\psi\right)+\frac{1}{2}V+\frac{3}{4\psi}\partial^c\psi\partial_c\psi\right)g_{ab};\end{aligned} \tag{13}$$

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$$(\varphi - \psi) G_{ab} = \kappa^2 T_{ab} + \frac{3}{2\psi} \partial_a \psi \partial_b \psi + \nabla_a \nabla_b (\varphi - \psi) - \\ - \left(\Box (\varphi - \psi) + \frac{1}{2} V + \frac{3}{4\psi} \partial^c \psi \partial_c \psi \right) g_{ab}; \tag{13}$$

$$\Box \psi - \frac{1}{2\psi} \partial^a \psi \partial_a \psi - \frac{1}{3} (V_\varphi + V_\psi) = 0; \tag{14}$$

$$\Box \varphi + \frac{1}{3} [2V - (\varphi - \psi + 1) V_\varphi - V_\psi] = \frac{\kappa^2 T}{3}. \tag{15}$$

FLRW universes

Friedmann-Lemaître-Robertson-Walker (FLRW) line element:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right] \tag{16}$$

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Friedmann equations:

$$3\left(H^2 + \frac{k}{a^2}\right) = \frac{1}{\varphi - \psi}\left[\kappa^2\rho + \frac{V}{2} + 3\left(\frac{\dot{\psi}^2}{4\psi} - H\left(\dot{\varphi} - \dot{\psi}\right)\right)\right], \tag{17}$$

$$2\left(\dot{H} - \frac{k}{a^2}\right) = \frac{1}{\varphi - \psi}\left[-\kappa^2(\rho + p) - \frac{3\dot{\psi}^2}{2\psi} + H\left(\dot{\varphi} - \dot{\psi}\right) - \left(\ddot{\varphi} - \ddot{\psi}\right)\right]; \tag{18}$$

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Klein-Gordon equations:

$$\ddot{\psi} + 3H\dot{\psi} - \frac{\dot{\psi}^2}{2\psi} + \frac{\psi}{3}\left(V_\varphi + V_\psi\right) = 0, \tag{19}$$

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{3}\left[2V - (\varphi - \psi + 1)V_\varphi - V_\psi\right] = \frac{\kappa^2}{3}(\rho - 3p). \tag{20}$$

Accelerating models

Variable transformation: $x = \log a$, which implies that $\dot{x} = \dot{a}/a = H$.

$$\frac{d}{dt} = H\frac{d}{dx} \qquad \frac{d^2}{dt^2} = \dot{H}\frac{d}{dx} + H^2\frac{d^2}{dx^2} \tag{21}$$

$$\psi_{xx} + \left(\frac{\dot{H}}{H^2} + 3\right)\psi_x - \frac{\psi_x^2}{2\psi} + \frac{1}{3H^2}\left(V_\varphi + V_\psi\right) = 0; \tag{22}$$

$$\varphi_{xx} + \left(\frac{\dot{H}}{H^2} + 3\right)\varphi_x - \frac{1}{3H^2}\left[2V - (\varphi - \psi + 1)\,V_\varphi - V_\psi\right] = \frac{\kappa^2}{3H^2}\left(\rho - 3p\right) \tag{23}$$

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Choosing a potential of the form $V = V_0\left(\varphi - \psi\right)^2$ implies

$$\psi_{xx} + \left(2 - q\right)\psi_x - \frac{\psi_x^2}{2\psi} = 0; \tag{24}$$

$$\varphi_{xx} + \left(2 - q\right)\varphi_x = 0. \tag{25}$$

Accelerating models

Another variable transformation: $u = \psi_x$ and $v = \varphi_x$.

$$\psi_{xx} = u\frac{du}{d\psi} \qquad \varphi_{xx} = v\frac{dv}{d\varphi}. \tag{26}$$

The equations simplify to:

$$u\frac{du}{d\psi} + (2-q)\,u - \frac{1}{2\psi^2}u^2 = 0; \tag{27}$$

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These equations can be solved for $\psi(a)$ and $\varphi(a)$:

$$\varphi(a) = \frac{C_1}{2-q} + C_2 a^{q-2} \quad ; \quad \psi(a) = \left[\frac{a^{q-2}e^{C_4(q-2)} + C_3}{2(2-q)}\right]^2. \tag{29}$$

Accelerating models

Solution for the scale factor:

$$-\frac{\dot{H}}{H^2} = \frac{d}{dt}\frac{1}{H} = q + 1 \implies a(t) = C_2 \left[(q+1)\,t + C_1\right]^{\frac{1}{q+1}} \tag{30}$$

Conditions:

1. $a(t = 0) = 0 \implies C_1 = 0$
2. $a(t = t_0) = a_0 \implies C_2 = a_0 \left[(q+1)\,t_0\right]^{-\frac{1}{q+1}}$

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The solution for the scale factor is

$$a(t) = a_0\left(\frac{t}{t_0}\right)^{\frac{1}{q+1}}. \tag{31}$$

In the particular case $q = -1$ the solution is different

$$\dot{H} = 0 \Leftrightarrow H = H_0 \Leftrightarrow a(t) = a_0 e^{H_0(t-t_0)}. \tag{32}$$

Accelerating models

Conditions for the scalar fields:

1. $\varphi(a=\infty)=\varphi_\infty$; $\psi(a=\infty)=\psi_\infty$;
2. $\varphi(a=a_0)=\varphi_0$; $\psi(a=a_0)=\psi_0$.

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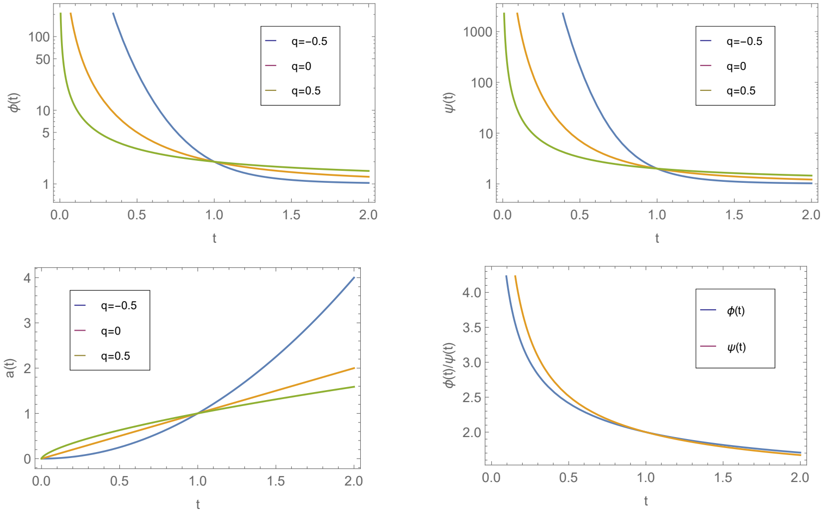
The solutions for the scalar fields become

$$\varphi(a)=\varphi_\infty\left[1+\left(\frac{a}{a_0}\right)^{q-2}\left(\frac{\varphi_0}{\varphi_\infty}-1\right)\right]; \tag{33}$$

$$\psi(a)=\psi_\infty\left[1+\left(\frac{a}{a_0}\right)^{q-2}\left(\sqrt{\frac{\psi_0}{\psi_\infty}}-1\right)\right]^2. \tag{34}$$

This analysis is only valid for $-1<q<2$.

Accelerating models



Conclusions

-Analytical solutions can be obtained in this theory;

-An accelerated expansion rate arises naturally;

-The potential that simplifies the equations has a physical meaning;

-The solutions decrease with an increase in the scale factor;

-More generalizations such as $R^{\mu\nu}R_{\mu\nu}$ terms can be considered;

-The theory also supports wormhole and black-hole solutions; (work in progress)

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Thank you for your attention!