Predictive Textures for the Neutrino Mass Matrix

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Outline

Neutrino Basics

- Neutrinos (ν) interact very weakly and only through the weak force and gravity.
- Neutrinos appear in three flavours, electron neutrinos (ν_e), muon neutrinos (ν_μ), and tau neutrinos (ν_τ).
- In the SM (only ν_L) neutrinos are strictly massless. No gauge invariant mass term ($\mathbf{Y}_\nu^{\alpha\beta}\,\overline{\ell_{L\alpha}}\widetilde{H}\,\nu_{R\beta} \rightarrow \mathbf{m}_\nu^{\alpha\beta}\,\overline{\nu_{L\alpha}}\nu_{R\beta}$), due to the absence of ν_R.
- Experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidences for the existence of neutrino oscillations.

Neutrino Oscillations

The fact that neutrinos oscillate imply non-vanishing masses and non-vanishing mixing angles.

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{2E}L\right)$$

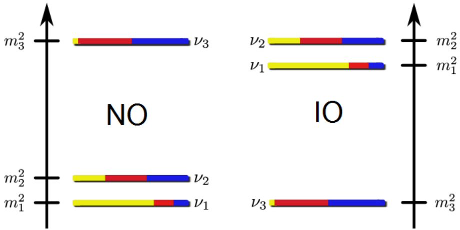
Like in the quarks case with the CKM matrix, for neutrinos, PMNS matrix represents the mismatch between mass states and interaction states that take part in the charged weak current.

$$\nu_i = \sum_j \mathbf{U}^{ij}_{PMNS}\,\nu_j,\;\; i = e, \nu, \tau,\;\; j = 1, 2, 3.$$

Neutrino Oscillations

The hints for the masses squared differences do not provide an absolute scale of neutrino mass and $0\nu\beta\beta$ experiments are yet to confirm the nature of neutrino: Dirac or Majorana particle.

Two possibles orderings:



Neutrino Oscillations

The hints for the masses squared differences do not provide an absolute scale of neutrino mass and $0\nu\beta\beta$ experiments are yet to confirm the nature of neutrino: Dirac or Majorana particle.

Dirac: $\mathbf{m}_\nu^{\alpha\beta}\,\overline{\nu_{L\alpha}}\,\nu_{R\beta} \rightarrow \mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu = \mathbf{d}_n = \mathrm{diag}\,(m_1, m_2, m_3)$
Majorana: $\mathbf{m}_\nu^{\alpha\beta}\,\overline{\nu_{L\alpha}}\,\nu_{L\beta}^c \rightarrow \mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu^* = \mathbf{d}_n = \mathrm{diag}\,(m_1, m_2, m_3)$

In the basis where charged lepton are diagonal ($\mathbf{m}_e = \mathbf{d}_e$) we have $\mathbf{U}_{PMNS} = \mathbf{U}_e^\dagger \mathbf{U}_\nu = \mathbf{U}_\nu$

Neutrino Oscillations

It can be parametrized (Standard parametrization) as $\mathbf{U}_\nu =$

$$\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \times$$

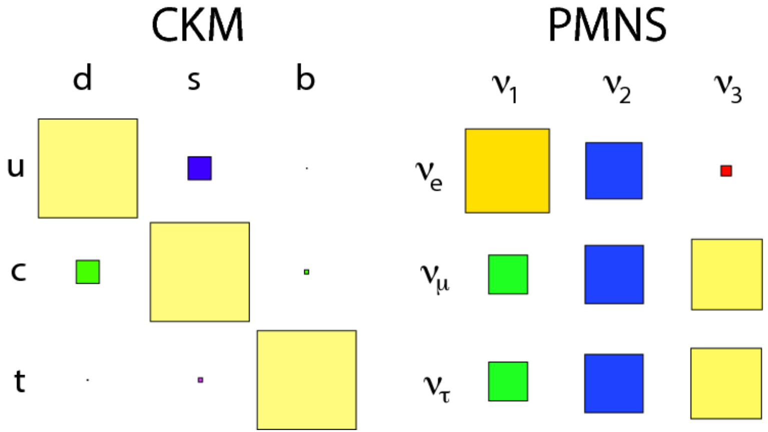
$\mathrm{diag}(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$,

where c_{ij} and s_{ij} are $\cos\theta_{ij}$ and $\sin\theta_{ij}$ respectively. The Dirac CP-violating phase is δ and the Majorana phases α_{21} and α_{31} are present only in the Majorana case.

Neutrino Oscillation Data

Parameter	Best fit $\pm 1\sigma$	3σ
Δm^2_{21} $[10^{-5}\,\text{eV}^2]$	$7.60^{+0.19}_{-0.18}$	7.11 – 8.18
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\,\text{eV}^2]$ (NO)	$2.48^{+0.05}_{-0.07}$	2.30 – 2.65
(IO)	$2.38^{+0.05}_{-0.06}$	2.20 – 2.54
$\sin^2\theta_{12}/10^{-1}$	3.23 ± 0.16	2.78 – 3.75
$\sin^2\theta_{23}/10^{-1}$ (NO)	$5.67^{+0.32}_{-1.24}$	3.93 – 6.43
(IO)	$5.73^{+0.25}_{-0.39}$	4.03 – 6.40
$\sin^2\theta_{13}/10^{-2}$ (NO)	2.26 ± 0.12	1.90 – 2.62
(IO)	2.29 ± 0.12	1.93 – 2.65
δ/π (NO)	$1.41^{+0.55}_{-0.40}$	0.0 – 2.0
(IO)	1.48 ± 0.31	0.0 – 2.0

Neutrino Oscillation Data



Neutrino Oscillation Data

Two additional constraints from experiments:

- The effective neutrino mass parameter, $m_{\beta\beta} = |\sum_i \mathbf{U}_{\nu\, ei}^2\, m_i|$, as it is related to the $\mathbf{U}_\nu$ data.
- The bound on neutrino mass sum, $\sum_i m_i < 0.23$ eV (95 % CL) obtained by the Planck mission, assuming three species of degenerate massive neutrinos and a ΛCDM model.

How can the data constrain the structure of the high energy Lagrangian, in particular for the neutrino mass matrix ($\mathbf{m}_\nu$)?

The Flavour Puzzle

There is no compelling theory to explain the origin of the lepton flavour structure. In the SM all mass matrices are free.

From the theoretical point of view, a natural approach is to restrict the number of free parameters in the lepton flavour sector so that the theory becomes more predictive.

Usual frameworks for the Majorana neutrino mass matrix ($\mathbf{m}_\nu$):

- Zero textures
- Hybrid textures

The Flavour Puzzle

In zero textures, zeroes may appear as a result of weak basis transformations or symmetries ($U(1)$, Z_4) constraints and they have been extensively studied. More than two-zero textures (five physical parameters) are ruled out.

$$T : \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \quad \longrightarrow \quad T^{1,2} : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad T^{4,6} : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

The Flavour Puzzle

Hybrid textures (five physical parameters), which have one texture zero and two equal nonzero elements, can also be implemented trough symmetries. They are well-motivated and discussed in literature since most of them are still compatible with current data.

$$T : \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} \quad \longrightarrow \quad T^6_{12} : \begin{pmatrix} X & X & * \\ X & * & * \\ * & * & 0 \end{pmatrix}, \quad T^3_{15} : \begin{pmatrix} X & * & 0 \\ * & * & X \\ 0 & X & * \end{pmatrix}$$

Motivation

To look for other neutrino textures which have the same predictability or which are even more predictive (contain less than five physical parameters, $n \leqslant 5$) and can accommodate the neutrino oscillation data.

$$n = 5 \longrightarrow \mathrm{T}_{134} : \begin{pmatrix} a & * & a \\ * & a & * \\ a & * & * \end{pmatrix};$$

$$n = 4 \longrightarrow \mathrm{T}^{2}_{34,56} : \begin{pmatrix} * & 0 & a \\ 0 & a & b \\ a & b & b \end{pmatrix}, \; n = 3 \longrightarrow \mathrm{T}^{2,3}_{145} : \begin{pmatrix} a & 0 & 0 \\ 0 & a & a \\ 0 & a & * \end{pmatrix}$$

Textures and Classes Identification

We denote the position of any vanishing element labeled i with a superscript, i.e. T^i, while equal elements with labels $i, j, \ldots, k$ are listed as subscripts, $\mathrm{T}_{ij\ldots k}$. For instance, the notations $\mathrm{T}^{5,6}_{12,34}$ and $\mathrm{T}^{3,4}_{15,26}$ correspond to the textures

$$\mathrm{T}^{5,6}_{12,34} = \begin{pmatrix} a & a & b \\ a & b & 0 \\ b & 0 & 0 \end{pmatrix} \quad \text{and} \quad \mathrm{T}^{3,4}_{15,26} = \begin{pmatrix} a & b & 0 \\ b & 0 & a \\ 0 & a & b \end{pmatrix}.$$

Textures and Classes Identification

$$\mathrm{T}^{5,6}_{12,34} = \begin{pmatrix} a & a & b \\ a & b & 0 \\ b & 0 & 0 \end{pmatrix}, \quad \mathrm{T}^{3,4}_{15,26} = \begin{pmatrix} a & b & 0 \\ b & 0 & a \\ 0 & a & b \end{pmatrix}$$

Each of these textures has two independent texture zeroes and two doublets, belonging to the same class $2_2 0_2$.

We classify different textures according to the number of correlated matrix elements using the notation N_{n_N} to specify that the matrix $\mathbf{m}_\nu$ contains N equal elements and that such correlation appears n_N times in the given texture. So, in general, we identify any given class of textures as $4_{n_4} 3_{n_3} 2_{n_2} 1_{n_1} 0_{n_0}$.

General Strategy

Use the MINUIT package to minimize the χ^2-function,

$$\chi^2(x) = \sum_i \frac{(\mathcal{P}_i(x) - \overline{\mathcal{O}}_i)^2}{\sigma_i^2}.$$

The $\mathbf{m}_\nu$ matrix elements are denoted by x, $\mathcal{P}_i(x)$ are the predictions of the *Ansätze* (each texture in each class) for the observables $\mathcal{O}_i$, $\overline{\mathcal{O}}_i$ are the best-fit values of $\mathcal{O}_i$, and σ_i are their corresponding $1\sigma/3\sigma$ errors.

A given pattern is only considered compatible at $1\sigma/3\sigma$ level if each observable (Δm^2_{ij}, θ_{ij}, and δ) fits in its $1\sigma/3\sigma$ interval.

Textures Results

n	Texture class	No. of textures	Solutions $(1\sigma, 3\sigma)$ NO	Solutions $(1\sigma, 3\sigma)$ IO
2	$4_1 0_2$	7^* $(\subset 15)$	-	-
3	$2_2 0_2$	21^* $(\subset 45)$	-	-
	$3_1 1_1 0_2$	28^* $(\subset 60)$	-	-
	$3_1 2_1 0_1$	60	(0,1)	(0,2)
	3_2	10	-	-
	$4_1 1_1 0_1$	30	-	-
	$4_1 2_1$	15	-	-
4	$2_1 1_2 0_2$	42^* $(\subset 90)$	(4,2)	(1,3)
	$2_2 1_1 0_1$	90	(15,3)	(19,11)
	$3_1 1_2 0_1$	60	(9,3)	(13,10)
	$3_1 2_1 1_1$	60	(13,5)	(13,5)
	$4_1 1_2$	15	(5,0)	(3,1)
5	$1_4 0_2$	7^* $(\subset 15)$	(6,0)	(3,2)
	$2_1 1_3 0_1$	60	(31,4)	(36,7)
	2_3	15	(4,0)	(1,1)
	$3_1 1_3$	20	(19,0)	(18,0)

Textures with Three Parameters

$$\text{NO} \longrightarrow \text{T}^{2}_{136,45} : \begin{pmatrix} a & 0 & a \\ 0 & b & b \\ a & b & a \end{pmatrix};$$

$$\text{IO} \longrightarrow \text{T}^{4}_{123,56} : \begin{pmatrix} a & a & a \\ a & 0 & b \\ a & b & b \end{pmatrix}, \quad \text{T}^{6}_{123,45} : \begin{pmatrix} a & a & a \\ a & b & b \\ a & b & 0 \end{pmatrix}.$$

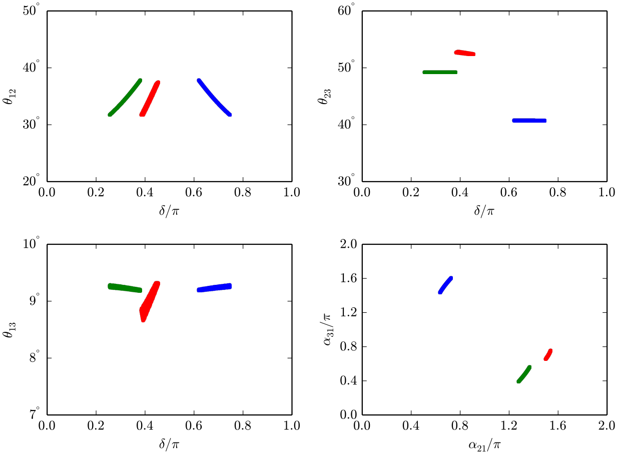
Textures with Three Parameters

Neutrino observables are quite constrained in the three cases, some can even be approximately predicted:

Spectrum	Texture	δ/π	m [meV]	$m_{\beta\beta}$ [meV]	α_{21}/π	α_{31}/π
NO	$T^2_{136,45}$	0.38 – 0.45	11 – 13	9.9 – 11	≈ 1.5	0.66 – 0.75
IO	$T^4_{123,56}$	0.26 – 0.38	9.7 – 11	27 – 30	1.3 – 1.4	0.39 – 0.56
	$T^6_{123,45}$	0.62 – 0.74	9.7 – 10	27 – 30	0.63 – 0.72	1.4 – 1.6

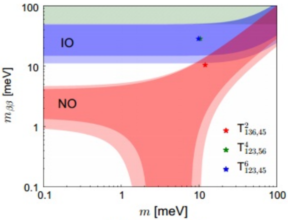
Since the complex matrix elements are free, there is an internal symmetry $\delta \to -\delta$, $\alpha_{21} \to -\alpha_{21}$ and $\alpha_{31} \to -\alpha_{31}$. This symmetry and the ones between $T^4_{123,56}$ and $T^6_{123,45}$ explain the very similar predictions.

Textures with Three Parameters

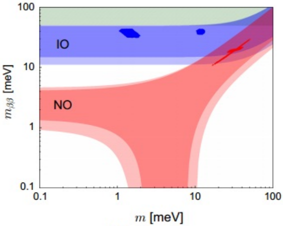


$T^2_{136,45}$ (NO) $T^4_{123,56}$ (IO) $T^6_{123,45}$ (IO)

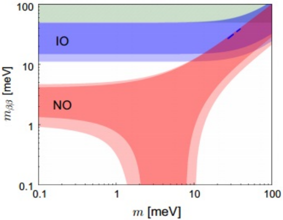
Effective Neutrino Mass Parameter Predictions



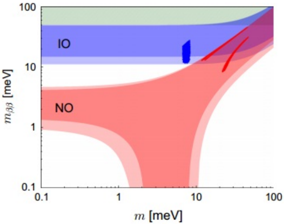
(a) Class $3_1 2_1 0_1$



(b) Class $4_1 1_2$



(c) Class $2_1 1_2 0_2$



(d) Class 2_3

Breaking Paths

If patterns with $n = 3$ can be implemented through symmetries, we may check whether they are or not compatible with data at the 1σ CL once the symmetries are (softly) broken.

Texture	$3_1 2_1 1_1$	$3_1 1_2 0_1$	$2_2 1_1 0_1$	$2_1 1_3 0_1$
$T^2_{136,45}$	$T_{136,45}$ (-)	T^2_{136} (-)	$T^2_{13,45}$ (NO) $T^2_{16,45}$ (NO) $T^2_{36,45}$ (-)	T^2_{45} (NO, IO)
$T^4_{123,56}$	$T_{123,56}$ (IO)	T^4_{123} (IO)	$T^4_{12,56}$ (IO) $T^4_{13,56}$ (IO) $T^4_{23,56}$ (IO)	T^4_{56} (IO)
$T^6_{123,45}$	$T_{123,45}$ (IO)	T^6_{123} (-)	$T^6_{12,45}$ (-) $T^6_{13,45}$ (-) $T^6_{23,45}$ (-)	T^6_{45} (IO)

Summary and Conclusions

- We randomize the elements of $\mathbf{m}_\nu$ for several textures with equal entries, then diagonalise through $\mathbf{U}_\nu^\dagger \mathbf{m}_\nu \mathbf{U}_\nu^*$ and check whether the observables reconstructed from $\mathbf{U}_\nu$ are compatible with data at $1\sigma/3\sigma$ CL.
- Many compatible classes of textures with increased predictability were found. Indeed, three textures with only three physical parameters tightly constrain the lightest neutrino mass, $m_{\beta\beta}$ and the complex phases (δ, α_{21} and α_{31}).
- An interesting possibility is to see if such predictive textures could arise from a symmetry principle, as well as test their stability under the renormalisation group evolution.
- Further studies and data are needed to discriminate, among the viable neutrino mass matrix textures, the patterns that more appropriately describe the observations.